Resources and Energy 4 (1982) 231–263. North-Holland Publishing Company

A SYSTEMMODEL OF COMBINED HEAT AND POWER GENERATION IN DISTRICT HEATING

Aviel VERBRUGGEN*

Universitaire Faculteiten Sint-Ignatius, 2000 Antwerp, Belgium

A model for evaluating the economic performance of cogeneration steam plants in district heating systems is discussed. Emphasis is on a detailed analysis of the production process. Analytical functions are presented for the main determinants of economic performance, i.e., heat load, temperature and mass flow of the water, heat and electric output and fuel consumption of the cogeneration units. Illustrative results, obtained with the model, are given.

1. Introduction

1.1. Low-temperature heat

In this paper low-temperature heat is energy transferred as heat at temperatures above the ambient temperature and below 120°–130°C (248°–266°F). This energy is seldom recognized as a separate product. The demand for low-temperature heat is met by the conversion of high-quality energy (fossil fuels, electricity) taking up 30 to 40% of the total energy flow in some industrialized nations [BMFT (1977)]. Large quantities of low-temperature heat are transmitted as waste to the environment. Markets for low-temperature heat were mostly not organized for simple reasons. Conveyance of low-temperature heat is expensive and the commodity is difficult to store in large amounts for longer periods. These properties require demand and supply to be geographically close and temporally synchronous. Because primary energy was cheap and environmental amenities were abundant, there was little incentive to recuperate low-temperature heat by complex conservation systems.

The 1973 energy price rise stimulated research on the organization of the low-temperature heat market, especially on industrial cogeneration and district heating [e.g. BMFT (1975), BMFT (1977)].

*I wish to thank W. Van Gool, editor of this Journal and two anynomous referees for their extensive comments on a previous draft. Thanks are also due to F. Verhaest, J.L. Roisin and P. Van Speybroeck for their support. P. Coene contributed a lot in improving the text by rewriting parts of it and by stimulating me to clarify some ideas.

1.2. District heating

District heating is the supply of low-temperature heat (viz. space heating and tap water heating) to urban areas by means of hot water at temperatures ranging from 80°C (175°F) to 130°C (266°F). In Europe, all modern district heating systems use pressurized hot water as heat carrier. Steam distribution is no longer considered because it requires high temperature levels (about 230°C, 450°F) making waste heat recovery difficult. The hot water is distributed through an insulated, underground, double-pipe network. Through one pipe the hot water is circulated from central plants to the customers, and through the other pipe the cooled water is pumped back to the central stations. The heat may be produced in heat-only plants or cogenerated with power, and the fuel can be anything from fossil or fissile to household or industrial waste [OECD (1978)]. In large district heating systems the bulk of the heat is generated in combined processes, where an important share of the distributed heat is recovered waste heat. The customer needs a suitable installation to transfer the heat from the distribution network into the hot water circuit of his building. Inside the living space, district heating resembles an oil or gas fired central heating system.

District heating is applied on a large scale in Sweden, Finland, Denmark, West-Germany and Eastern Europe. In other European countries (e.g. the Netherlands, Belgium, United Kingdom, France) the market share of district heating is very small. In these countries, project appraisal studies are undertaken and some district heating systems are built.

1.3. Scope and outline of the paper

District heating is a complex undertaking. For its practical analysis one will decompose this problem in several parts, e.g., demand estimation, distribution network optimization, heat production optimization, etc. In this paper, attention is focused on the construction of a model for production optimization. The main part of the model carries on evaluation of the economic performance of cogeneration units. In section 2, an overview of the production planning problem is given, indicating the limitations of our approach.

Section 3 described the operation modes of district heating systems. A mathematical description of the main modes is derived to test their impact on the cogeneration costs.

The central part of the paper is an analysis of the cogeneration process (section 4). Cogeneration technology is represented in a new way, permitting to study in depth the impact of the main design parameters of the district heating system and of the cogeneration units on the cost of cogenerated heat. The set of equations derived in sections 3 and 4 provides a valuable tool for district heating planning, and particularly for choosing the optimum cogeneration unit in existing district heating systems. In practice, a district heating system grows gradually. During the first years of network construction, heat is generated in boiler plants or recovered in facilities if these are available (domestic refuse incineration plants, industries). After a few years the decision to build a specially designed CHP station is taken. Engineering firms and turbine manufacturers bid for the order and the DH company has to select one offer. For this decision, it is necessary that the company can compare in detail the economic impact of at first glance minor differences in the technical characteristics of the proposed units. Because the model discussed in this paper incorporates these technical details, it is suited for carrying out these comparative computations.

The cost of cogenerated heat depends on the value assigned to the cogenerated electricity. This problem of allocating the joint costs to the outputs is examined in section 5.

In section 6 a few results are presented to illustrate the impact of the main decision variables, and some concluding remarks are summarized in section 7.

2. Composition of a DH production park

The planning of heat generation in DH systems is similar to the planning of electricity production [Turvey and Anderson (1977)]. Storage of heat and electricity for a long period and on an extensive scale is not economical. Demand for both products varies heavily over time and can be represented by means of load duration curves. In each case, production plants of diverse technologies are installed to minimize generation costs. Therefore, the methodology developed for electricity planning is also used in heat planning. Some modifications, however, are necessary because of the particularities of the cogeneration processes and because of the characteristics of low-temperature heat.

First, the load duration curve of heat demand is presented. In the remainder of this section, the basic concepts of heat production planning are reviewed to provide a reference framework for the model developed later on.

2.1. *Heat load*

The annual demand for heat at the production facilities of a large district heating system is usually represented by means of a load duration curve [BMFT (1977), Frilund (1975)]. This curve represents for each hour of the year the heat demand, ordered from highest to lowest value (see fig. 1). The heat load duration curve is thus an ordered set of 8760 pairs (t_i, l_i), t_i being the hours of the year and l_i the corresponding heat loads (in MW), such that if $t_j > t_i$, then $l_j \leqq l_i$.

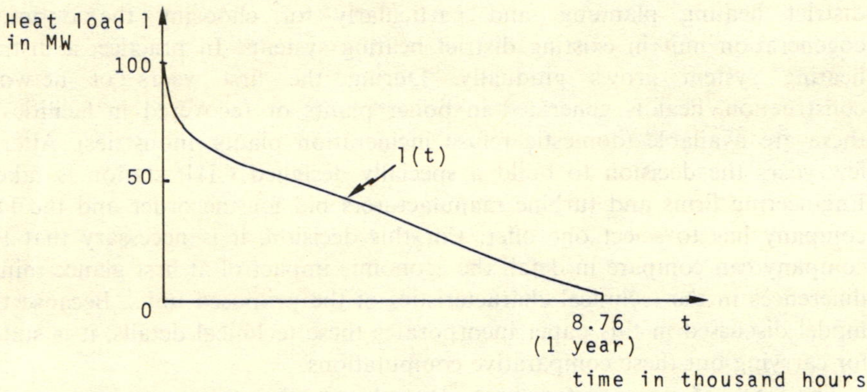

Fig. 1. Standard heat load duration curve.

A standard load duration pattern has been estimated for Belgium [Verbruggen (1980)] and is shown in fig. 1. It has been observed that this pattern is very similar all over Europe, not withstanding the significant differences in climate [AGFW (1981), BMFT (1977), Frilund (1975), Verbruggen (1980)].

It is cumbersome to handle vectors of 8760 elements. Therefore, an analytical expression of the load duration curve is estimated by Lagrange's collocation polynomial method [Scheid (1968)]. To obtain an accurate representation of the curve in fig. 1, we estimate three segments separately. For a heat load duration function scaled on a peak load $l_{\max}=100$ MW, we obtained the following function ($t=1000$ hours):

$$l_1(t)=100.0-178.0355t+424.0144t^2$$

$$-467.2733t^3+184.6676t^4, \qquad 0\leqq t\leqq 1,$$

$$l_2(t)=73.9697-11.0344t-0.1872t^2+0.8246t^3$$

$$-0.2114t^4+0.0151t^5, \qquad 1\leqq t\leqq 6,$$

$$l_3(t)=-237.4391+127.1106t-19.5142t^2$$

$$+0.9282t^3, \qquad 6\leqq t\leqq 8.76, \tag{1}$$

In the sequel, this function (1) will be called $l(t)$. The load pattern represented by (1) implies an utilization of 3240 hours per annum (i.e., a load factor of 37%). Function (1) can be used for any project scale after adapting the coefficients of the polynomial (e.g., all coefficients are multiplied by 10 if the peak load equals 1000 MW instead of 100 MW).

2.2. Production planning

The purpose of production planning is to determine the type and capacity of the available technologies in order to minimize total production costs subject to the constraint that the demand $l(t)$ is met. DH costs are primarily heat delivery costs (transportation, distribution, heat transfer at the customer's buildings) and heat generation costs. In a planning study the two components can, to a great extent, be analyzed separately. However, the interaction between both is crucial in the operation of a DH network, fixing inter alia the temperature levels of the distributed heat (section 3) and the location of the production facilities. In this article, we consider only the production side of the problem but allow for the effects of the production model on the distribution planning, i.e., the operation mode of the network will be parameterized in the production model.

A two-step procedure is used to determine the development of a production park. First, we derive the optimum long-run composition of the park (the masterplan). This is obtained by minimizing total annual production costs when the project is finished. This problem is examined in this paper. The development of the system from start to masterplan is not discussed here, because it depends on several specific variables, e.g., network construction, growth of the heat load, availability of particular heat supplies [Verbruggen (1979)]. The two-step procedure is a second-best approach. The difference between the result obtained with this approach and an overall optimum depends on the time horizon of the analysis, the time preference of the decision-maker and the quality of the models used in the two approaches. The procedure used here is also used in Germany [BMFT (1977)].

CHP production involves the joint generation of heat and electricity. Therefore, one should plan the heat generation system and the electricity generation system simultaneously. In practice it is difficult to attain this ideal, because of lack of data with respect to the demand for both products, particularly with respect to the synchronism of the demand patterns. In addition, because of the large difference in conveyance costs of heat and electricity, heat demand is locally limited while the market for electricity is nation-wide or even transnational. Consequently, it becomes even more difficult to determine the synchronism in both demand patterns. However, given the extent of the electricity market and the limited scale of the electricity cogenerated in a DH system one can safely assume that this power can always be sold to the electric grid. This allows for a separate planning of heat and electricity production. In planning DH production, electricity is considered as a by-product evaluated at its opportunity cost, i.e., the cost of producing electricity in a large efficient system (see section 5). Such an assumption was also used in BMFT (1977).

The remainder of this section contains a brief description of DH production planning methodology which is, as mentioned above, similar to the methodology of electricity planning [Turvey and Anderson (1978)].

Assume there are n technologies available, with annual per unit (MW) capacity costs K_i, and per unit (MWh) operating costs M_i. Assume, moreover, that none of the technologies is dominated, i.e., if $M_j < M_i$ and $K_j > K_i$, then

$$M_i + K_i < M_j + K_j \quad \text{and} \quad 8760\ M_i + K_i > 8760\ M_j + K_j$$

ensures that the two technologies i and j break even at a particular utilization time,

$$t^*_{i,j} = (K_j - K_i)/(M_i - M_j).$$

For a utilization less than $t^*_{i,j}$, technology i is the best one, and for a utilization exceeding $t^*_{i,j}$ hours, j is the best. Or, in general, if t_i is the utilization time of technology i, then the least-cost arrangement of the n technologies implies

$$M_j < M_i \rightarrow t_j > t_i.$$

In order to find the lowest operating cost of the system, the technologies are ranked from lowest to highest M_i, i.e., the merit-order of the park is

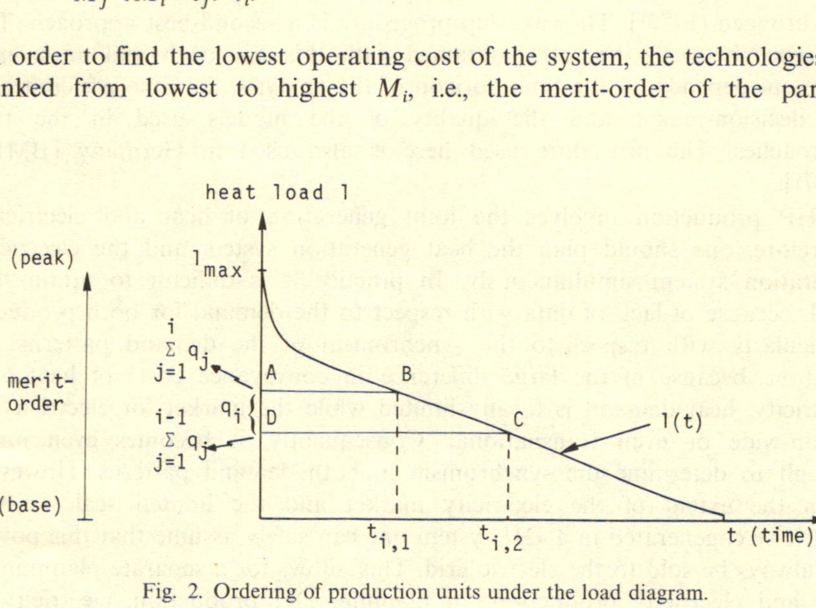

Fig. 2. Ordering of production units under the load diagram.

constructed. This merit-order is then placed on the ordinate axis of the load diagram in order to quantify the production costs (see fig. 2). Base-load units (lowest M_i, highest K_i) are placed at the bottom of the load diagram, and so on, up to peak load units (highest M_i, lowest K_i).

The result for a particular technology i is illustrated in fig. 2. The planned capacity of this technology is q_i MW, and its production is projected to cover the area *ABCD* of the load surface. During $t_{i,1}$ hours of the year, unit i is fully loaded and during $(t_{i,2}-t_{i,1})$ hours it is partly loaded.

The planning problem can now be stated formally as

$$\min_{q_1,\ldots,q_i,\ldots,q_n} \sum_i K_i * q_i$$

$$+\sum_i M_i * \left\{ q_i * t_{i,1} + \int_{t_{i,1}}^{t_{i,2}} 1(t)\,\mathrm{d}t - [t_{i,2}-t_{i,1}] * \sum_{j=1}^{i-1} q_j \right\} \tag{2}$$

(sum of capital and operating costs),

$$\text{subject to} \quad q_i \geqq 0, \quad \sum_i q_i \geqq 1_{\max}, \qquad i=1,\ldots,n,$$

where the expression in { } is the production delivered by unit i (area *ABCD*).

The planning problem becomes more complex when the assumption of linear technologies is relaxed, i.e., when K_i and/or M_i are functions of q_i and possibly of the place of unit i under the load diagram (t_{i1}, t_{i2}).

Several approaches can be used to solve this planning problem. The interested reader is referred to the excellent survey by D. Anderson [Turvey and Anderson (1978)]. In this study, a direct approach is employed. That is, the load duration curve $l(t)$ is incorporated in the objective function. Such an approach is feasible for DH planning since the number of available technologies is limited. It also allows for a detailed analysis of the cogeneration process as will be shown below.

The non-dominated technologies in district heating systems are primarily: large-scale hot water boilers as peak and reserve units,[1] cogeneration plants, and specific waste heat recovery facilities. For cogeneration, steam-cycle plants (back-pressure and extraction-condensing), combined gas–steam plants, gas turbines and combustion engines (diesels and gas-engines) can be considered. In large DH systems, coal-fired steam-cycle units dominate the

[1]The provision of reserve capacity is not discussed in this article. In the computer program, a subroutine program for calculating the loss-of-load-probability of the production park is provided. If this computed LOLP value is higher than some target LOLP, reserve capacity is added. The target LOLP value represents the required quality of service. The optimal service quality depends on factors that are difficult to quantify [Telson (1975)].

other CHP technologies that use expensive high-quality fuels (natural gas, oil) and involve high maintenance costs. The current analysis will be limited to back-pressure and extraction-condensing units, although the same methodology can be applied to the other cogeneration technologies [Verbruggen (1979)].

Waste heat recovery facilities are specific for each DH system. The availability of domestic refuse incineration facilities or of industries offering low-temperature heat (e.g., oil refineries), is an input to the DH planning process. It turns out that such waste heat capacity is used as base-load. Since the availability of waste heat capacity is case dependent, it is not incorporated in this study.

Summarizing, the technologies which will be considered are hot water boilers, extraction-condensing steam turbines and back-pressure steam turbines. This limitation simplifies our problem and is not inappropriate for large-scale DH systems.

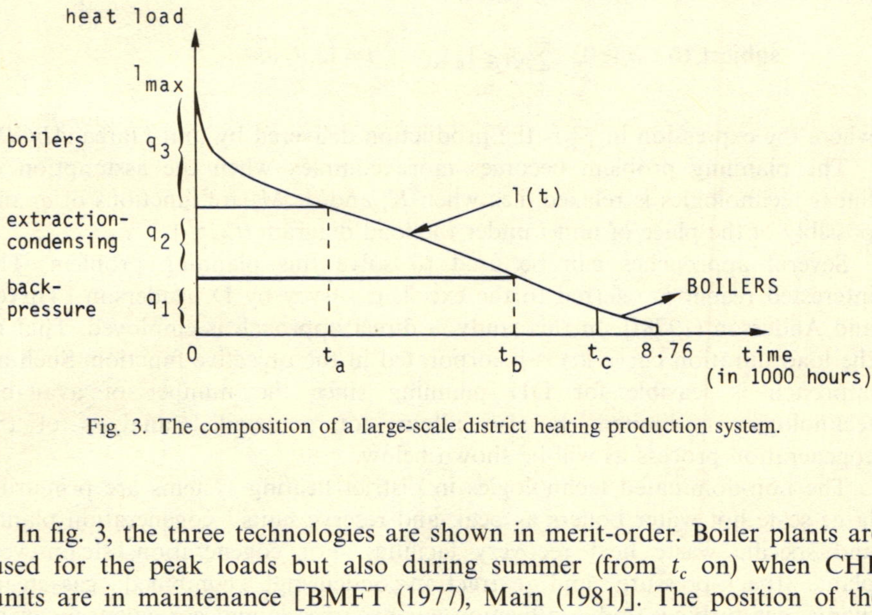

Fig. 3. The composition of a large-scale district heating production system.

In fig. 3, the three technologies are shown in merit-order. Boiler plants are used for the peak loads but also during summer (from t_c on) when CHP units are in maintenance [BMFT (1977), Main (1981)]. The position of the extraction-condensing and the back-pressure capacities is at this stage of the discussion difficult to explain, because it depends on the method of allocating the joint costs over heat and electricity (section 5). Roughly stated, the position of the back-pressure turbines at the bottom of the heat load diagram follows from the complementarity of heat and electricity generation in the back-pressure process.

The thus simplified planning problem is represented by the following optimisation problem:

$$\min_{q_1,q_2,q_3} K_3(q_3)+M_3(q_3,t_a)*\left\{\int_0^{t_a} l(t)\,\mathrm{d}t-(q_1+q_2)*t_a\right\}$$

$$+M_3(q_3,t_c)*\left\{\int_{t_c}^{8.76} l(t)\,\mathrm{d}t\right\}+K_2(q_2)$$

$$+M_2(q_2,t_a,t_b)*\left\{q_2*t_a+\int_{t_a}^{t_b} l(t)\,\mathrm{d}t-q_1*(t_b-t_a)\right\}-\mathrm{Re}(q_2,t_a,t_b)$$

$$+K_1(q_1)+M_1(q_1,t_b,t_c)*\left\{q_1*t_b+\int_{t_b}^{t_c} l(t)\,\mathrm{d}t\right\}-\mathrm{Re}(q_1,t_b,t_c), \tag{3}$$

[sum of the three expressions for boilers ($i=3$), extraction-condensing ($i=2$) and backpressure ($i=1$)],

$$\text{subject to} \quad q_i \geqq 0, \quad \sum_i q_i \geqq l_{\max}, \qquad i=1,2,3,$$

where Re() represents the revenue from the sale of cogenerated electricity. The functions $K_i()$ and $M_i()$ are the capacity cost and fuel cost functions of the three technologies.

To implement (3), we need to estimate the functions $M_i()$, $K_i()$ and Re(). This issue is discussed in sections 4 and 5. Before our analysis of the CHP technology we first give a strict definition of the examined operation modes of a DH system in section 3.

Anticipating our results, we examine how the least-cost masterplan (q_1^*, q_2^*, q_3^*) is determined. In a simple grid search procedure, the objective function (3) is evaluated for several values (q_1^j, q_2^j, q_3^j) and the minimum is retained as the optimum solution. The procedure tends to converge in a small number of iterations when the composition of existing DH production systems is used as starting point. For example, when $0.4 \leqq (q_1^\circ + q_2^\circ)/l_{\max} \leqq 0,6$, and q_3° is the residual share of $l_{\max}$. The optimum assignment of the common capacity $(q_1^j + q_2^j)$ to back-pressure and extraction-condensing capacity requires a more careful analysis because several local minima can occur. In general there are at least three local minima: at 100% of $(q_1^j + q_2^j)$ in back-pressure, at 100% of $(q_1^j + q_2^j)$ in extraction-condensing, and somewhere in between these extremes. However, in large DH systems several interior local minima may be present due to economies of scale and indivisibilities in the available technologies. Therefore, a fine grid search over $(q_1^j + q_2^j)$ is necessary.

The approach of optimizing a DH production park by evaluating a large set of (q_1^j, q_2^j, q_3^j) solutions is feasible when the computing time of each evaluation is small. Because the model derived below is a set of analytical functions, the evaluation of a particular solution requires little time.

3. The operation of district heating systems

The planning of a system is affected by the way the system will be operated. Several operation modes of DH networks are feasible. To enable the choice of the optimum one, their impact on the final cost balance should be quantified. An operation mode is determined by the temperature levels of the circulating water mass at any moment and by the way the temperature of this mass is raised from level $T_r(t)$, the temperature of the water arriving at the central production plants, to $T_v(t)$, the temperature of the water leaving these plants.

With respect to the former determinant two different modes are common. In the first one, the temperature of the leaving water $T_v(t)$ remains constant all over the year, or in general, is set at a constant level $T_{v,max}$ during winter and at a constant level $T_{v,min}$ during summer. This principle is common practice in plants without cogeneration. CHP generation gave rise to the second mode. In this case, the temperature of the leaving water flow $T_v(t)$ is lowered gradually during winter from the maximum value $T_{v,max}$ at heat peak load conditions to the minimum level $T_{v,min}$ in the summer. The operation mode is called 'gliding temperature operation' and is defined strictly in section 3.1. It allows to keep the $T_{v,max}$ temperature level quite high, so that the diameters (viz. investments) of distribution network pipes can be kept smaller. At the same time it allows for a decrease in the temperature of the heat exhausted at the cogeneration units.

To raise the temperature from $T_r(t)$ to $T_v(t)$, the production units can be connected in parallel or in series. In the first case, all facilities work in parallel each one of them heating the water from $T_r(t)$ to $T_v(t)$. In the second case, units are placed in a serial arrangement each one of them increasing the water temperature over a subinterval of the range $T_r(t) \rightarrow T_v(t)$. In section 3.2, an arrangement is discussed connecting the CHP units in series with the peak load units.

3.1. Mass flow and temperature of district heating water

An energy amount transferred as heat by means of a fluid is proportional to the product of the mass flow circulated and the difference between outgoing and returning fluid temperatures. Formally,

$$l(t) = c \cdot V(t) \cdot \{T_v(t) - T_r(t)\}, \tag{4}$$

with

c = proportionality factor,
$V(t)$ = mass flow of hot water at time t,
$T_v(t) - T_r(t)$ = temperature difference at time t,
$l(t)$ = heat load at time t.

A target heat load $l(t)$ can be met by an infinite number of combinations of water flow and temperature difference.

Most DH systems are operated with gliding outgoing temperatures $T_v(t)$, i.e., during winter the outgoing water temperature reaches a maximum $T_{v,max}$ at heat peak load conditions, and then decreases to some minimum $T_{v,min}$ in summer. During the period of gliding, the water mass pumped around remains constant at its maximum flow V_{max}. In other words, target heat load $l(t)$ is met by varying the temperatures of the water while keeping the mass flow constant. During summer, $T_v(t)$ is held constant at $T_{v,min}$ and the mass flow is diminished with lower heat loads $l(t)$.

Assume $T_r(t) = T_r$ at every moment, because $T_r(t)$ varies little over the year [BMFT (1977), Mean (1981)].

We now derive expressions for the outgoing temperature $T_v(t)$ and the mass flow $V(t)$ as functions of the heat load function $l(t)$ and the parameters T_r, $T_{v,max}$ and $T_{v,min}$.

At heat peak load ($t = 0$, see fig. 1), $l(0) = 1_{max}$, $T_v(0) = T_{v,max}$, and also $V(0) = V_{max}$. Substituting in eq. (4) results in

$$c \cdot V_{max} = l_{max}/(T_{v,max} - T_r). \tag{5}$$

For $t < t^*$, where t^* is the value of t for which the minimum admissible outgoing temperature $T_{v,min}$ is reached (t^* is derived below), the system is operated with gliding $T_v(t)$ temperatures and constant mass flow V_{max}. Using eqs. (4) and (5), it follows that for $0 \leqq t \leqq t^*$:

$$T_v(t) = [(T_{v,max} - T_r)/l_{max}] \cdot l(t) + T_r, \tag{6}$$

$$V(t) = V_{max}. \tag{7}$$

In other words, the temperature function $T_v(t)$ is a linear transformation of the load duration function $l(t)$.

When $t^* \leqq t \leqq 8.76$ (t is in 1000 hours),

$$T_v(t) = T_{v,min}. \tag{6'}$$

and using (4), it follows that

$$V(t) = [1/c(T_{v,min} - T_r)] \cdot l(t). \tag{7'}$$

t^* is the switching point between the 'winter' operating regime ($t<t^*$) and the 'summer' regime ($t>t^*$). It is easy to see that t^* is the solution of eq. (6) where $T_v(t)$ is replaced by $T_{v,min}$. Formally,

$$l(t^*)=[(T_{v,min}-T_r)/(T_{v,max}-T_r)]\cdot l_{max}. \tag{8}$$

The three functions $l(t)$, $T_v(t)$ and $V(t)$ are shown in fig. 4. The parameters of expressions (6) and (7), T_r, $T_{v,min}$ and $T_{v,max}$, can be chosen by the

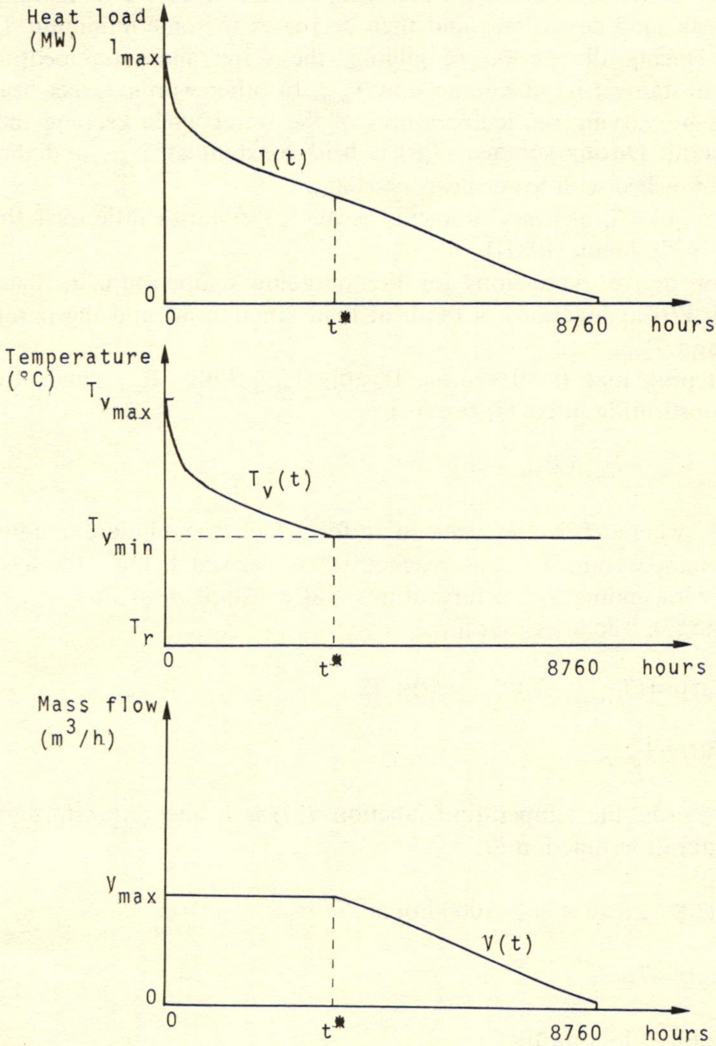

Fig. 4. Heat load, water temperatures and mass flow in a DH system; gliding temperature operation.

producer. The system of equations derived above allows to evaluate the effects of the choice on these parameters, on the overall cost of the cogeneration system through their effects on $T_v(t)$ and $V(t)$ (see section 4.1).

3.2. Stepwise heating of the DH water

The amount of electricity generated in the CHP plant increases significantly by the gliding temperature operation. Not only electric production over the year but also the electric capacity available at any moment is important. Even with gliding temperature operation, electric capacity available is lowered sharply during heat peak load hours. Unfortunately, in Belgium these heat peak load hours tend to correspond to the hours of electric peak load demand. To reduce the loss of electric production capacity during winter, stepwise heating of the water can be set up.

In this configuration (shown in fig. 5), the temperature of the returning DH water is raised in several steps during the coldest hours of the year: first, from T_r to some $TA < T_v(t)$ in the CHP plant and secondly from TA to $T_v(t)$ in a boiler peak plant. For $t > t^0$, where t^0 is defined by $T_v(t^0) = TA$, peak heating is no longer required and the operation of the system is the classical gliding temperature mode. Assuming that $TA > T_{v,\min}$ it follows from the definition and from (6) that t^0 is determined by

$$l(t^0) = (TA - T_r) \cdot l_{\max}/(T_{v,\max} - T_r). \tag{8'}$$

Stepwise heating is only practical if CHP plants are built in the neighbourhood of the district so that high temperature differences are not needed to reduce heat transport costs [BMFT (1977)]. When stepwise

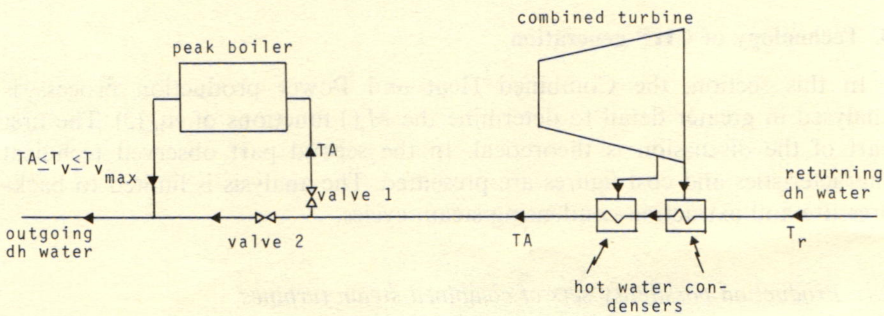

Fig. 5. Heating of the district heating water in series. Winter exploitation: valve 1 opens while valve 2 is closed correspondingly; summer exploitation: valve 1 closed.

heating is applied, a specific function is assigned to the base and peak load units. For a particular set (q_1^j, q_2^j, q_3^j) of planned capacities (see section 2), TA can be derived from

$$(q_1^j + q_2^j)/l_{\max} = (TA - T_r)/(T_{v,\max} - T_r). \tag{9}$$

Eq. (9) states that TA should be set at a level such that, at heat peak load CHP capacity raises the temperature of all the water flow $V_{\max}$ from T_r to TA and peak capacity raises it from TA to $T_{v,\max}$. If TA would be set higher than its value resulting from (9), it would follow that stepwise heating is applied to only part of the system and one could gain by lowering TA. On the other hand, if TA is lower than its value resulting from (9), the provided peak capacity is inadequate to raise the temperature of the water from this TA to $T_{v,\max}$.

Stepwise heating raises the electricity production of the CHP turbines and ensures a higher electric production capacity during wintertime. Although the deviation from parallel heating lasts only during the coldest hours of the year, the gain in electric production may be large. Lower extraction temperatures at the turbines, and more constant working conditions for the units permit a better design of the turbines [Frilund (1975), Mühlhäuser (1977), Zenker (1981)]. When TA is low (as it usually is), two hot water condensers can ensure high power yields. Assume, e.g., $T_{v,\max} = 120°C$, $T_r = 60°C$ and $q_1^j + q_2^j = q_3^j = \frac{1}{2} l_{\max}$ (i.e., a common configuration observed in existing systems), then from (9) it follows that $TA = 90°C$. With (8′) and (1) we find $t^o = 2694$ hours. This means that during about one third of their operation time, invariable load conditions are imposed on the turbines. These conditions are good candidates to figure as the design conditions of the units [Mühlhäuser (1977), Zenker (1981)]. In addition, because the gap $TA - T_r = 30°C$, two hot water condensers each bridging half of this gap will be provided [Frilund (1975)].

4. Technology of CHP generation

In this section, the Combined Heat and Power production process is analysed in greater detail to determine the $M_i()$ functions of eq. (3). The first part of the discussion is theoretical. In the second part observed technical characteristics and cost figures are presented. The analysis is limited to back-pressure and extraction-condensing steam cycles.

4.1. Production possibility sets of combined steam turbines

By virtue of the first law of thermodynamics, and considering the turbine of a CHP steam cycle as the system under study, the amount of energy transferred to the system as heat (q_B) is equal to the sum of the energy flow delivered as work (e) plus the energy flow exhausted from the turbine to the condenser as heat (q). Assuming that thermal radiation losses and mechanical losses remain constant, these energy flows can be left out of the argument here.

When the live steam flow q_B to the turbine remains constant, and the temperature T_r of the DH water entering the CHP plant remains also constant, with most CHP turbines (equipped with two or more hot water condensers) it is feasible to raise the temperature of the DH water flow from T_r to a wide range of temperatures $T > T_r$. To fix the ideas, assume that the temperature of the water flow is raised from T_r, once to 90°C and once to 120°C. Then one can state that

$$q_B(90°\mathrm{C}) = q_B(120°\mathrm{C})$$

because in the two states the supply of live steam flow remains constant, or

$$e(90°\mathrm{C}) + q(90°\mathrm{C}) = e(120°\mathrm{C}) + q(120°\mathrm{C})$$

by virtue of the first law of thermodynamics, or

$$e(90°\mathrm{C}) - e(120°\mathrm{C}) = q(120°\mathrm{C}) - q(90°\mathrm{C}),$$

or

$$\Delta e = -\Delta q. \tag{10}$$

Expression (10) states that an electric production capacity slice Δe is replaced by a heat production capacity slice Δq of the same magnitude when the temperature of the steam exhausted at the turbine is increased. This obvious conclusion will be incorporated in the analysis in a while. But first we have to examine another technical relationship of, e.g., a back-pressure turbine.

The production possibility set of a back-pressure turbine is usually described by a curve or bundle of curves, representing the power yield, i.e., the ratio e/q, as a function of the percentage load on the unit, i.e., e/e_{max}. Each curve corresponds to different pressure and temperature characteristics of the live steam and exhausted steam flow. One such curve is shown in fig. 6a. If the non-linear curve is mapped into an (E, Q) plane, it turns out that the possibility set can be represented by a line segment (fig. 6b). Because it is easier to work with linear relationships, the remainder of the analysis is based on the format of fig. 6b.

The combination of (10) and fig. 6b results in the diagram shown fig. 7a. In the latter picture, the line segment XW is taken from fig. 6b. XW represents the (E, Q) production possibilities of a back-pressure turbine when the DH

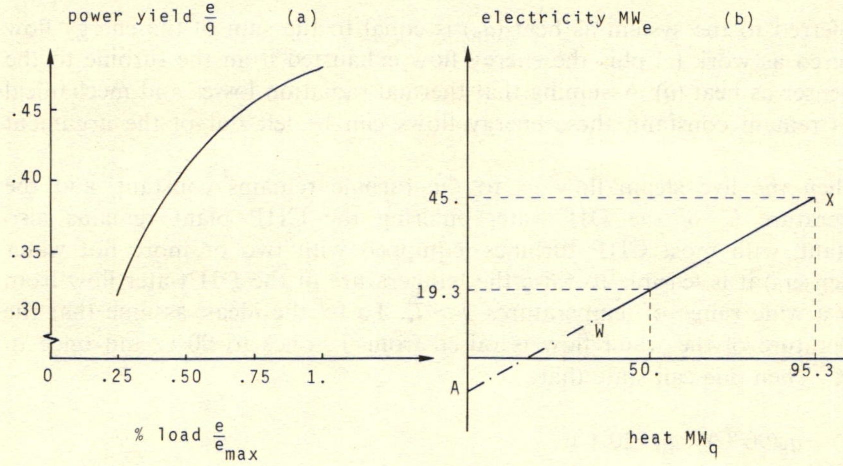

Fig. 6. Production possibility sets of a single exhaust 45 MW_e back-pressure turbine (live steam characteristics 105 bar/525°C, raising the temperature of the DH water flow from 60°C to 90°C). *Source:* Stal-Laval Turbine AB, Sweden.

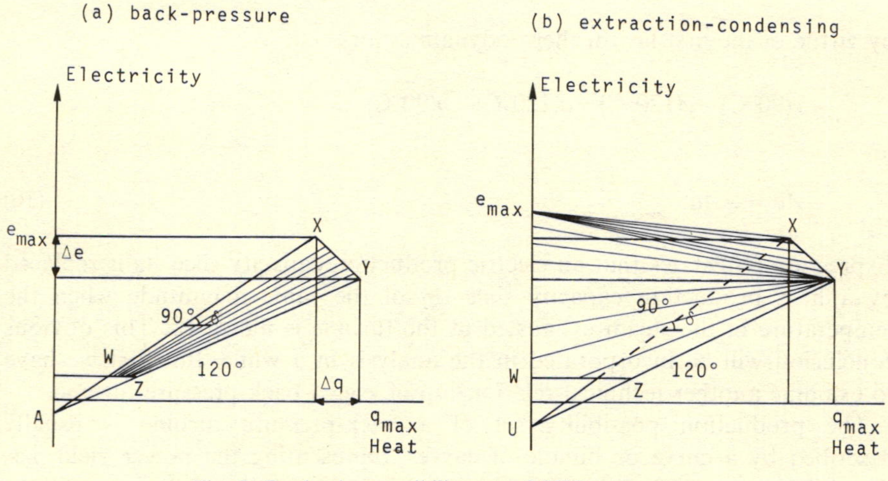

Fig. 7. Production possibility sets of CHP steam units.

water temperature is raised from T_r to 90°C. If this temperature is raised from T_r to 120°C the production possibility set is given by YZ. The position of Y is derived from the position of X, lowering the electric capacity of the turbine by Δe and increasing the heat capacity by Δq [see expression (10)]. The slope of the YZ line is determined by the position of Y and the virtual point A, being a technical characteristic of the turbine. Point A can be found by mapping a power yield curve in the (E, Q) plane (see fig. 6).

Applying the above argument to any temperature level between 90°C and

120°C, one obtains a bundle of lines between AX and AY. Consequently, the complete production possibility set of the back-pressure turbine is given by the area $XYZW$. Area AWZ is not included in the set because it is physically impossible to run a turbine below particular loads.

The heat production capacity of a back-pressure steam turbine depends on the temperature of the exhausted steam. The capacities are the abscissas of points on the segment XY in fig. 7a. The relation between capacities can be stated more formally by

$$q(T)=\Delta q\cdot(T-90)/30+q(90°\mathrm{C}), \tag{11}$$

where

$q(T)$ = heat production capacity of the turbine when the temperature of the water flow is raised to T,

$q(90°\mathrm{C})$ = idem, but where $T=90°\mathrm{C}$,

Δq = $q(120°\mathrm{C})-q(90°\mathrm{C})$, i.e., the increase in heat capacity when T increases from 90°C to 120°C.

Eq. (11) is the result of a linear interpolation in the interval XY. Considering the Mollier diagram for steam, this interpolation is a fair approximation of reality for the interval of the turbine expansion curve examined here [Reynolds and Perkins (1977)].

Before continuing the analysis, the production possibility set of extraction-condensing turbines is discussed. The type of plants examined here are specially built CHP units which can be operated in quasi back-pressure mode. When operated in this way, only a minor part of the steam is sent to the low-pressure turbine blades in order to prevent their heating up [Frilund (1975), Mühlhäuser (1977)]. The cold-condensing tail of the unit is dimensioned in order to optimize the cogeneration process. One should be aware of the difference between this type of plants and conventional cold-condensing steam plants. In the latter plants the amount of steam that can be extracted is limited, because otherwise mechanical distorting forces would damage the turbine.

The production-possibility set of the extraction-condensing unit is shown in fig. 7b. Area UXY corresponds to the quasi back-pressure operation mode of the unit and is derived in the same way as the AXY area of fig. 7a. The feasible (E,Q) set of an extraction-condensing turbine is given by area $WZYXe_{\max}$ because of the availability of a cold-condensing tail. Line $Xe_{\max}$ corresponds to a full load operation of the unit. Along this line more and more power is substituted for exhausted heat as $e_{\max}$ is approached. The segment WZ shows the lower bound of the load for a safe operation of the turbine. The heat production capacity of the extraction-condensing turbine is also given by eq. (11).

4.2. Heat generation

The heat generated by a CHP plant, represented by $Q(T,t)$, depends on the temperature T to which the DH water flow has to be raised by the plant from level T_r, and on t, indicating the actual position of the unit in the load diagram (see fig. 3). In section 3 it has been shown that the temperature T depends on the mode of operation of the system, and consequently T depends also on t. For example, if stepwise heating is applied, then

$$T(t) = TA, \qquad 0 < t \leqq t^{o},$$

$$T(t) = [(T_{v,\max} - T_r)/l_{\max}] \cdot l(t) + T_r, \qquad t^{o} < t \leqq t^{*},$$

$$T(t) = T_{v,\min}, \qquad t^{*} < t \leqq 8.76. \tag{12}$$

When substituting (12) for T in (11) one finds the capacity of a CHP unit when stepwise heating is applied. Putting $t^{o} = 0$, one obtains the heat capacity of the unit when stepwise heating is not used.

The amount of heat energy delivered by a CHP unit used at capacity is derived by integrating the heat capacity functions of the CHP unit over the respective time intervals. When there is no full heat load on the unit the generated heat is obtained by integrating the load duration function $l(t)$.

4.3. Electricity generation

To determine the revenue from the cogenerated electricity [function Re() in expression (3)], one first computes the amount of electricity generated by the CHP unit. For a given level of T, the production possibility set of a back-pressure turbine is a line (see e.g. fig. 6b), and the production possibilities of an extraction-condensing turbine can be viewed as a bundle of rays starting at a point on the vertical axis in the interval $U - e_{\max}$ and converging at a point (determined by T) on the segment XY (see fig. 7). Therefore, the electricity generated by a CHP unit is given by

$$E(T,t) = A + b(T) \cdot Q(T,t), \tag{13}$$

with

$Q(T,t)$ = heat generation at moment t, when the exhausted steam raises temperature of the DH water to T,

$b(T)$ = slope of an (E,Q) production-possibility line as a function of T (see fig. 7),

A = fixed technical parameter if the CHP unit is a back-pressing turbine, and $U \leqq A \leqq e_{\max}$ if it is an extraction-condensing turbine (fig. 7),

$b(T)$ = a linear function of T [Verbruggen (1979)].

Then

$$b(T)=m+n\cdot T, \tag{14}$$

where

$$n=-D/(1+D)\cdot(\mathrm{tg}\delta+1)/30, \tag{15}$$

$$m=(\mathrm{tg}\delta+4D\cdot\mathrm{tg}\delta+3D)/(1+D), \tag{16}$$

where $D=[q(120°\mathrm{C})-q(90°\mathrm{C})]/q(90°\mathrm{C})$ is the percentage increase (decrease) in heat (electricity) capacity if the temperature at the turbine exhaust is 120°C rather than 90°C, and where tgδ is the slope of the back-pressure operation (E, Q) possibility line when $T=90$°C (fig. 7).

Substituting (15) and (16) into (14), and (14) into (13) provides an analytical function of the electricity production at monent t when heat generation equals $Q(T,t)$ and the DH water temperature at the exhaust of the turbine is T.

For the back-pressure units, the electric capacity $e(T,t)$ available at t is given by (13), because the possibility of generating electricity depends proportionally on the heat load. In the case of an extraction-condensing turbine, (13) represents the available electric capacity when the term A in (13) equals $e_{\max}$, i.e., the maximum electric output of the unit at zero heat load.

4.4. Fuel consumption

In a last step, we determine the fuel consumprion of the CHP units. For a back-pressure unit this is quite simple, because of the complementarity of heat and power generation. The fuel consumption is given by

$$F(T,t)=(Q(T,t)+E(T,t))/\eta(t), \tag{17}$$

where $\eta(t)$ is the energy conversion efficiency of the unit.

For an extraction-condensing turbine the problem is more complex because of the substitutability between heat and electricity generation and because of the large difference in the energy-conversion efficiency of heat generation versus power generation.

Fig. 8 shows how fuel consumption depends on electricity generation and low-temperature heat output in an extraction-condensing plant [Frilund (1975), Mühlhäuser (1977)]. The production possibility set of the turbine is given by triangle DBC. Line DC corresponds to the cold-condensing operation, and line DB to the quasi back-pressure operation of the unit. Heat production is measured on the upper horizontal axis from the right corner

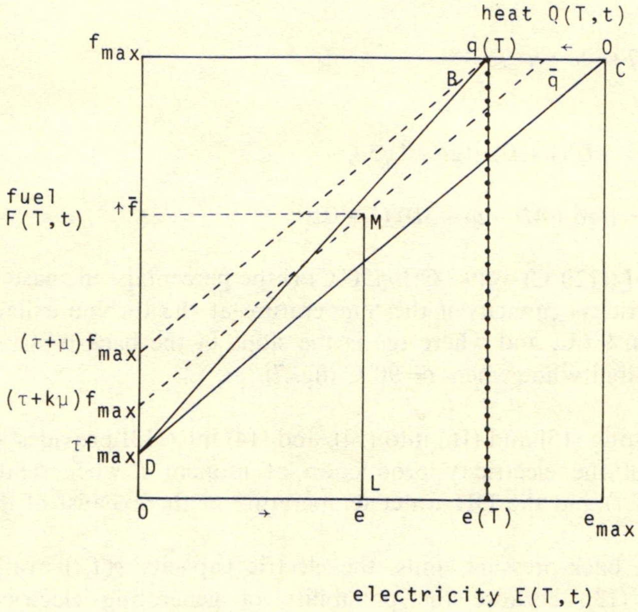

Fig. 8. Extraction-condensing steam plant; relation between heat output, electricity output and fuel consumption.

point C (heat output$=0$) to point B [maximum heat output $q(T)$, see expression (11)]. Electricity generated is represented by the lower horizontal axis from $0 \rightarrow e_{\max}$ ($e_{\max}$ corresponds to the maximum load at pure cold-condensing operation). Fuel consumption is indicated on the left vertical axis from $0 \rightarrow f_{\max}$. The point D corresponds to 'zero load steam flow' with a fuel requirement of $\tau \cdot f_{\max}$. The scaling on the three axis is different.

Fuel consumption is equal to the vertical dimension of the intersection between a line parallel to DC through $Q(T,t)$ on the upper horizontal axis, and a vertical line through $E(T,t)$. This is illustrated in fig. 8. The fuel needed to produce $\bar{q}$ units of heat and $\bar{e}$ units electricity is $\bar{f}$, or the length of the line segment ML.

The slope of $M\bar{q}$ and of the lines parallel to it equals that of DC, given by

$$(1-\tau)f_{\max}/e_{\max}. \tag{18}$$

The intercept of $M\bar{q}$ is given by $(\tau+k\mu)f_{\max}$, where μ denotes the additional use of fuel relative to $f_{\max}$ when for a constant production of electricity, heat production is increased from 0 to $q(T)$,

$$\mu=(1-\tau)(\{e_{\max}-e(T)\}/e_{\max}), \tag{19}$$

and k is the heat load fraction

$$k = Q(T,t)/q(T). \tag{20}$$

Combining (18), (19) and (20) gives

$$F(T,t) = \left[\tau + (1-\tau)\frac{e_{\max} - e(T)}{e_{\max}} \frac{Q(T,t)}{q(T)} + (1-\tau)\frac{E(T,t)}{e_{\max}} \right] \cdot f_{\max}. \tag{21}$$

Eq. (21) allows one to compute accurately fuel consumption as a function of heat and electricity load for an extraction-condensing turbine with zero load steam flow τ, electric capacity in pure cold-condensing mode of $e_{\max}$ and maximum heat load electric capacity $e(T)$. Dividing both sides of expression (21) by $f_{\max}$ one obtains the percentage fuel consumption as a function of the percentage heat load and the percentage electric load.

4.5. Data on CHP plants

This concludes the derivation of formal expressions of the energy flows of a CHP process. To implement these functions, the technical characteristics of the CHP units, the design parameters of the DH system (temperature levels T_r, $T_{v,\max}$, $T_{v,\min}$) and the operation mode of the system must be known.

Table 1
Characteristics of CHP extraction-condensing units (coal-fired).

Power range (MW_e)	60–150	150–250
Live steam characteristics (bar/°C)	105/520	180/535/535
Slope of the (E, Q) possibility line operating in back-pressure mode and raising the temperature of the DH water from 60°C to 90°C	0.478	0.628
Percentage increase in heat capacity when the above temperature levels are raised to 60°C/120°C	5%	5%
Zero load steam flow (τ in %)	6%	4%
Efficiency (pure cold-condensing)	32.3%	36.3%
Efficiency (maximum heat load)	88.7%	82%
Forced outage rate	8%	8%
Investment cost on 1/1/1981 ($/$kW_e$)[a]	690–640	690–640
Operating staff (number of workers in 6 shifts)	48	48

[a] 1 US \$ ≈ 40 Belgian francs in 1981.

Table 2

Characteristics of CHP back-pressure units (coal-fired).

Power range (MW_e)	2–60	60–120
Live steam characteristics: pressure (bar)	80–105	110–140
temperature (°C)	495–530	530–535
Slope of the (E, Q) possibility line raising the temperature of the DH water from 60°C to 90°C	0.472	0.546
Percentage increase in heat capacity when the above temperature levels are raised to 60°C/120°C	5%	5%
Efficiency at full load	85%	86%
Forced outage rate	6%	6%
Investment cost on 1/1/1981 ($/kW$_e$)[a]	610–590	610–590
Operating staff (number of workers in 6 shifts)	36	36

[a]1 US $ ≈ 40 Belgian franks in 1981.

To that end, data from existing CHP plants in Sweden and Germany and from turbine manufacturers (primarily from Stal-Laval Turbin AB) were gathered. The averages for the main characteristics are shown in table 1 for extraction-condensing plants, and in table 2 for back-pressure plants. See also Frilund (1975) and Mühlhäuser (1977).

4.6. Use of the results

A recapture of the main lines of the argument may be fruitful. Our analysis starts with the load duration function $l(t)$. First, the common operation modes of DH systems are defined strictly (section 3), resulting in functions for the temperature $T_v(t)$ of the outgoing DH water flow and for the mass flow $V(t)$ through the DH network.

From $T_v(t)$, one derives the function $T(t)$, the temperature level to which the DH water has to be raised at the CHP turbines. $T(t) = T_v(t)$ when no stepwise heating is applied. Otherwise, $T(t) = TA < T_v(t)$ during winter, and $T(t) = T_v(t)$ in the remainder of the year. Next, we analysed the CHP production process, deriving equations for heat and electricity output and fuel consumption. Heat output $Q(T, t)$ is either $q(T)$, i.e., the heat capacity of the unit, or is derived from $l(t)$ in the case of part load operation. Electric output $E(T, t)$ is estimated on the basis of $Q(T, t)$ and $T(t)$. Finally, fuel consumption $F(T, t)$ is derived from $Q(T, t)$ and $E(T, t)$. All of these functions are either linear or polynomials in t.

The above set of functions is organized in a computer program. Function subroutines for $l(t)$ and for all other functions derived from it are provided. The parameters of the transformations are placed in COMMON statements. A special subroutine QEF organizes the use of all these functions, calling in the appropriate expressions in function of the time interval of the year under study, and integrating them numerically over the interval. The QEF subroutine is called by the main program providing the technical characteristics of the analyzed CHP unit in the CALL statement and receiving the energy flows added over the year. If necessary, detailed results are printed by the QEF subroutine. In its organizing task, the QEF routine has to solve a polynomial equation repeatedly [e.g., to find t^* in expression (8), but primarily to find out the respective activity zones of the CHP units in the load diagram — see fig. 3]. This problem is solved by calling subroutine SN. In SN the inverse of $l(t)$, i.e., $l^{-1}(t)$ or $t(l)$, is incorporated.

It is out of scope of this article to give a complete description of the computer program that contains a lot of other options and subroutines. Evaluation of a particular project proposal consists of the projection of the masterplan, resulting from a grid search over the composition of the production park, and a dynamic recovery of this long-run optimal solution from the start of the project on. The latter is used as basis for a cash-flow analysis of the project. For details the reader is referred to [Verbruggen (1979)].

Because the computer program makes use of analytical functions, evaluating a proposal requires little computing time. Therefore, the program can be used as an interactive system model in DH project appraisal studies. As such, it is a valuable tool in a 'Decision analysis' cycle for district heating [Matheson and Howard (1968)].

In section 6, a few examples of selected results obtained with the program are examined, but first we have to consider in the next section the method used for allocating the joint costs of the cogeneration process.

5. Allocation of the joint cogeneration costs over heat and electricity

In this section the principle for determining the value of the cogenerated electricity is explained [see function Re in the objective function (3)]. This again is a rather complex problem and the discussion is limited to the main line of argument.

Let C_E be the cost of generating a particular quantity of electricity E in an electricity only plant, C_Q the cost of generating a particular quantity of heat Q in a heat only plant, and CC the cost of generating both E and Q in a cogeneration plant. Then, the feasibility of cogeneration requires

$$CC < C_E + C_Q.$$

Assuming that this condition is met, how should one value the projected electric output in a DH planning study? Theoretically there are an infinite number of assignments of CC to E and Q, represented by the points of line KL in fig. 9. Only a subset of these assignments is acceptable for both parties (electricity and heat), viz. the points of contract zone MN where the reference

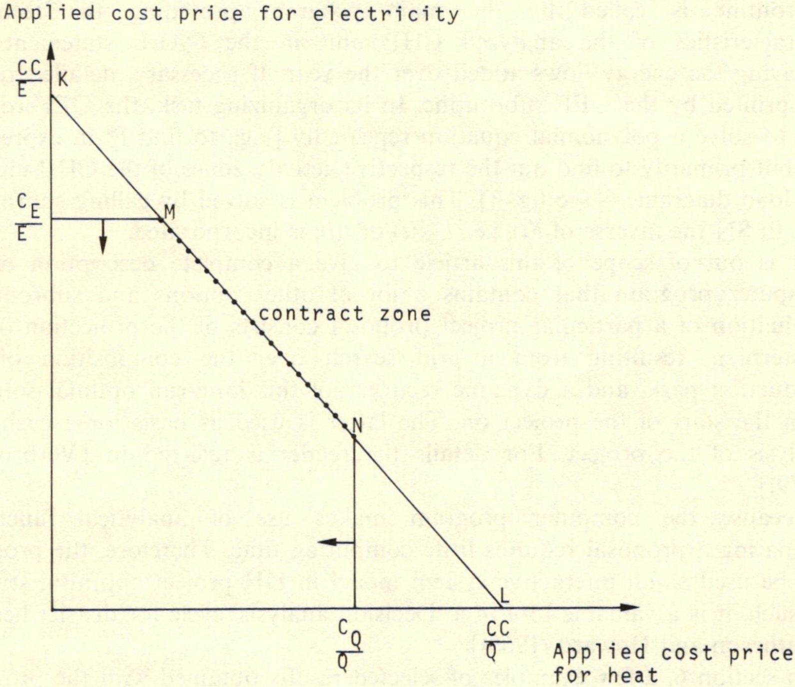

Fig. 9. Allocation of the joint CHP costs to heat and electricity.

cost price for electricity is lower than C_E/E and the one for heat is lower than C_Q/Q. All points of MN are feasible and that is still an infinite number.

The particular solution is determined by bargaining. In Belgium, this bargaining process is arrived at a stage where the electric utilities agree on the allocation represented by point M in fig. 9. This means that the utilities agree to buy the cogenerated electricity at a price equal to the cost of generation in their electricity only park. They accept that cogeneration will be a neutral activity for the electric grid. To guarantee this neutrality, cogenerated electricity should be valued at its opportunity cost. It is obvious that the rule also implies that all the gains of CHP generation are assigned to the DH system.

At this stage of the discussion, the problem is reduced to determining the opportunity cost of the cogenerated electricity. One should remember that the study is carried out in a planning perspective. Neutrality for the electric utilities in this perspective implies that they are indifferent between expanding their own electric-only system and contracting the electric part of the CHP system. Therefore, the opportunity cost equals the expansion cost or long-run marginal cost of the electric-only system. Because CHP plants are scheduled to work during many hours of the year (see fig. 3) they will

Table 3
Characteristics of electric base-load plants (Belgium, 1981).

	Coal	Nuclear
Power range (MW_e)	600	1300
Investment ($/$kW_e$)	540	1000
Staff & maintenance cost ($/$kW_e$-year)	16	26
Efficiency (%)	38	32
Fuel cost (mills/kWh_e)	21.08	8.45

provide base load electricity. Consequently, the cogenerated electricity should be valued at the long-run marginal cost of electric only base-load capacity. To implement the above principle one needs the characteristics of the plants considered by the electric utilities for base-load capacity (see table 3).

A few comments are in order. First, evaluating the opportunity cost as proposed, requires knowledge of the CHP electric production capacity guaranteed to the electric utilities and of the amount of electricity generated with it. The Re () function in (3) is obtained by multiplying the guaranteed capacity with the fixed cost of base-load electric plants and the generated electricity with the fuel cost of these plants, and by adding both results. CHP electric capacity is guaranteed when it is available on demand by the electric grid during any moment of the year. Adjustments for divergences in forced outage rates and total disposability of the plants under study should be made, because CHP capacity has a higher disposability than nuclear or coal-fired base load capacity. One can now understand why back-pressure capacity is planned at the bottom of the heat load diagram. Because a back-pressure unit can guarantee electric production capacity only when there is a heat load, the capacity cost of back-pressure units turns out, after correcting for the revenue from guaranteed electric capacity, to be higher than the capacity cost of extraction-condensing units, that can guarantee a high electric capacity all year. Second, the allocation rule is easy to implement. The cost figures used (table 3) are present-day, well-known magnitudes. Third, incentives to managerial efficiency are built in. The electric utilities are in last instance not affected by the construction of CHP plants and continue optimizing their production system as they did without CHP plants. The DH planner, on the other hand, has an incentive to install the best suited CHP technology.

The method proposed above is useful in organizing the discussion on the allocation issue, but it does not exclude the necessity for bargaining. As mentioned before, one has to bargain for a point of the contract zone. In addition, it requires a lot of bargaining to determine the opportunity cost of electricity generation. As shown in table 3, two electricity generation technologies are considered for expansion of the base-load: coal-fired and nuclear condensing plants. After selection of one of both (or combinations of them), one has to fix the operating conditions (e.g. forced outage rates of all plants considered, electric load target for the extraction-condensing turbines if no maximum heat load is available, etc.). Results of a limited sensitivity analysis with respect to this problem is discussed in the next section, because the Re( ) function in (3) is affected by the terms of the joint cost allocation contract.

6. Results

In this section a few results obtained with the model discussed above are presented. In section 6.1 we illustrate the sensitivity of the CHP costs for the projected operation mode of the DH system, and in section 6.2 the impact of the allocation of the joint costs is examined. The CHP process under study is imbedded in the analysis of a DH project with a yearly heat demand of 1500 GWh (5400 TJ). Production capacity consists of 180 MW_q reserve boilers, 257 MW_q peak boilers and 257 MW_q cogeneration units. In the coal-fired CHP plant, an extraction-condensing unit, or an extraction-condensing plus back-pressure unit are installed depending on the result of the least-cost search. The electricity opportunity costs are those of nuclear base-load condensing plants and the operating conditions are the worst a DH operator can expect (e.g. the extraction-condensing plants have to generate all year around their maximum electric output, see section 6.2).

6.1. Impact of the operation mode of the DH system on the long-run average cost of cogenerated heat, net of revenue from the cogenerated electricity

An operation mode of a DH system is defined by the design temperatures of the network ($T_{v,max}$, $T_{v,min}$, T_r) and by the way the production units are connected (stepwise or no-stepwise heating of the water flow). A sensitivity analysis is carried out. The temperature of the outgoing water during summer $T_{v,min}$ is fixed at 90°C and the other variables are allowed to vary. Results are summarized in figs. 10a and 10b. In fig. 10a, the costs of the CHP generation are considered when the electricity consumption for pumping the water through the DH network is fixed at the amount of the reference point ($T_{v,max} = 130$°C, $T_r = 70$°C and no-stepwise heating). In this case, electricity for pumping amounts to 38.6 GWh/year. In fig. 10b, the more realistic case of varying pumping costs in function of the design parameters of the system is examined.

The results shown in fig. 10a are clear. The cost of cogenerated heat decreases with decreasing temperature T_r of the water flow arriving at the CHP turbines. The lower the condensing temperature of the steam expanding in the turbine can be made, the higher is the efficiency of a Rankine cycle.

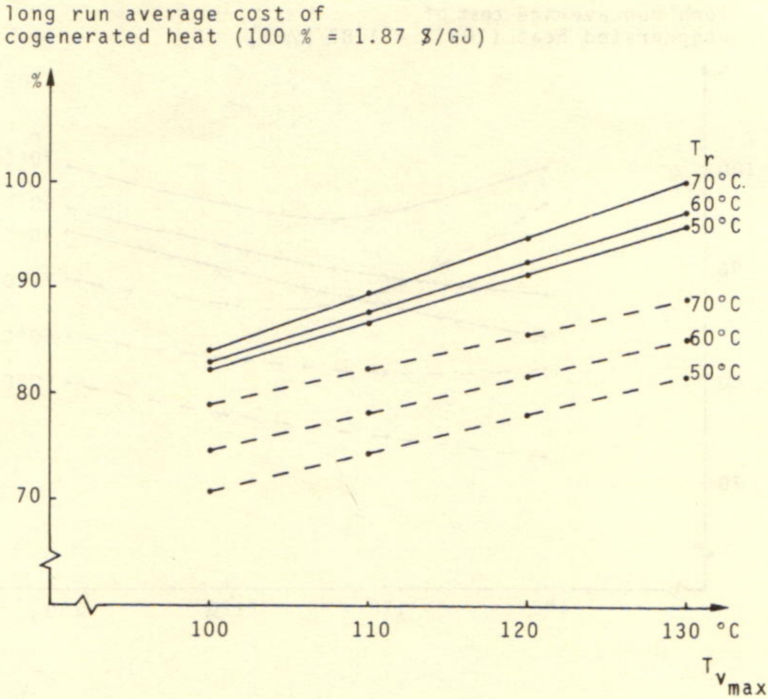

Fig. 10a. Long-run average cost of the cogenerated heat in function of the operation mode of the DH system — fixed electricity consumption for pumping. No-stepwise heating —— and stepwise heating ----. T_r = constant temperature of the DH water arriving at the CHP plants and $T_{v,max}$ = temperature of the outgoing DH water flow at heat peak load.

For the same reason, the curves shown in fig. 10a are upward sloping when $T_{v,max}$ is raised from 100°C to 130°C. The difference between stepwise heating (dashed curves) and no-stepwise heating (solid curves) is also due to the same physical laws governing the performance of steam turbines. The instructive message of the results of fig. 10a is contained in the slopes of the curves shown and in the distance between these curves. First, one should note the significant cost reduction when stepwise heating is applied. Second, the impact of T_r is important, i.e., incentives should be given to the customers to extract the maximum amount of energy out of a particular mass of hot water. This can be done, e.g., by billing heat (partly or entirely) on the basis of the mass flow of hot water passing through the customer's installation. Third, stepwise heating reduces the sensitivity of cogeneration costs to changes in $T_{v,max}$.

The cogeneration plant is, however, just one component of the DH system. Distribution operation and distribution investment costs are as important. Unfortunately, the latter costs increase when $T_{v,max}$ is reduced, because distribution pipes of larger diameter have to be constructed and more tons of

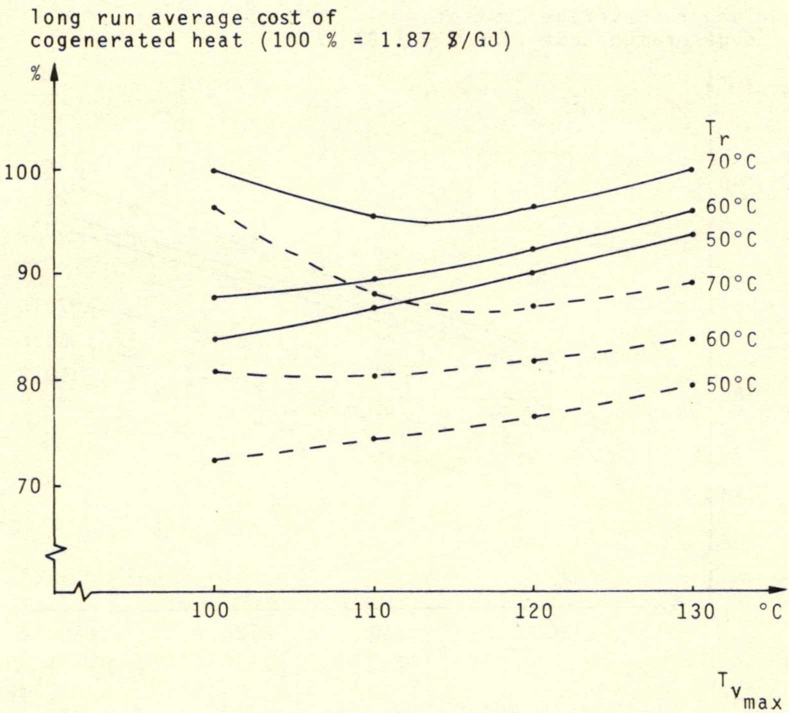

Fig. 10b. Long-run average cost of the cogenerated heat in function of the operation mode of the DH system — varying electricity consumption for pumping. No-stepwise heating —— and stepwise heating ----. T_r = constant temperature of the DH water arriving at the CHP plants and $T_{v,\max}$ = temperature of the outgoing DH water flow at heat peak load.

water have to be pumped around to meet the same heat load. In fig. 10b results are shown when allowance is made for the variation in pumping consumption. We still observe the distinction between stepwise and no-stepwise heating, the former always resulting in lower costs than the latter for particular values of T_r and $T_{v,\max}$. There is little interference between distribution on the one hand and the way of connecting the production units on the other hand.

For $T^r = 70°C$, the optimal $T_{v,\max}$ is no longer the lowest one, because electricity consumption for pumping increases rapidly when $T_{v,\max} - T_r$ becomes too small. For lower T_r, this effect is weakened. The significance of T_r is illustrated well in fig. 10b. It is clear that one must incorporate distribution costs in the calculations used to determine T_r and $T_{v,\max}$ [Van Speybroeck (1980)]. This requires a complete project appraisal of the DH system and exceeds the limits of this article. On the basis of the few results shown here, one can argue qualitatively for a serious effort to lower T_r and for stepwise heating of the water flow. If these two advices are followed, one

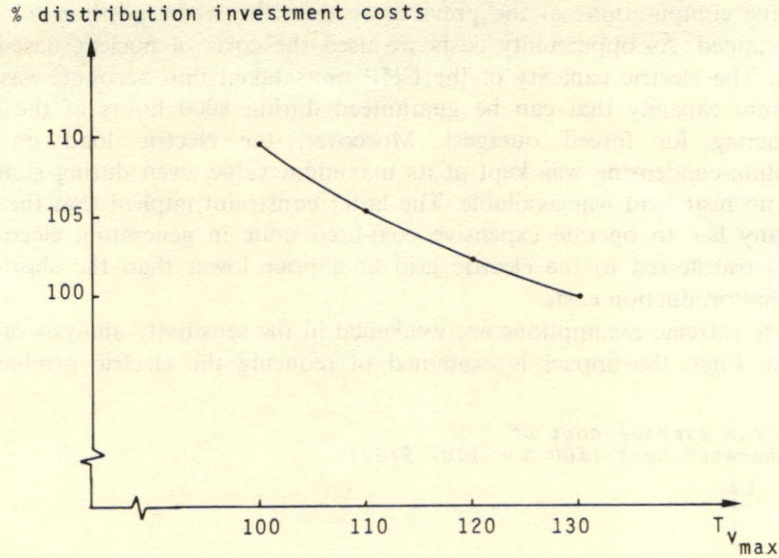

Fig. 11. Distribution investment costs in function of the maximum temperature of the water flow, $T_{v,max}$ ($T_r = 60°C$). *Source:* Van Speybroeck (1980).

is on the lowest curve in fig. 10b. The optimal value of $T_{v,max}$ will depend on the distribution investments. The latter decrease with higher $T_{v,max}$. An example, shown in fig. 11, is taken from a particular case study [Van Speybroek (1980)]. Of course, the optimal $T_{v,max}$ depends on the relative weight of distribution vis-à-vis cogeneration costs in the project under study.

6.2. *Impact of the opportunity cost of the cogenerated electricity on the long-run average cost of cogenerated heat, net of revenue from the cogenerated electricity*

As discussed in section 5, the opportunity cost of the cogenerated electricity is measured by the long-run marginal costs of base-load condensing capacity. It was also mentioned that the practical implementation of this principle requires bargaining between electric utilities and DH planners. One can argue that the transfer price for the cogenerated electric energy, should be equal to the fuel cost of planned base-load condensing capacity. Difficulties arise in determining the value of the electric capacity of the CHP plants. This capacity is not constant over time, but depends on the heat load on the CHP turbines (complementarity for back-pressure units and substitutability for extraction-condensing units). Moreover, the effective production capacity of a condensing plant and of a CHP plant depends on the expected forced outage and on the planned unavailability for maintenance, repair and fueling (nuclear plants).

In the computations of the previous section, the worst position for DH was retained. As opportunity costs we used the costs of nuclear base-load plants. The electric capacity of the CHP units taken into account, was the minimum capacity that can be guaranteed during 8000 hours of the year (abstracting for forced outages). Moreover, the electric load on the extraction-condensing was kept at its maximum value, even during summer when no heat load was available. The latter constraint implied that the DH company has to operate expensive coal-fired units in generating electricity that is transferred to the electric grid at a price lower than the short-run marginal production costs.

These extreme assumptions are weakened in the sensitivity analysis of this section. First, the impact is examined of reducing the electric production

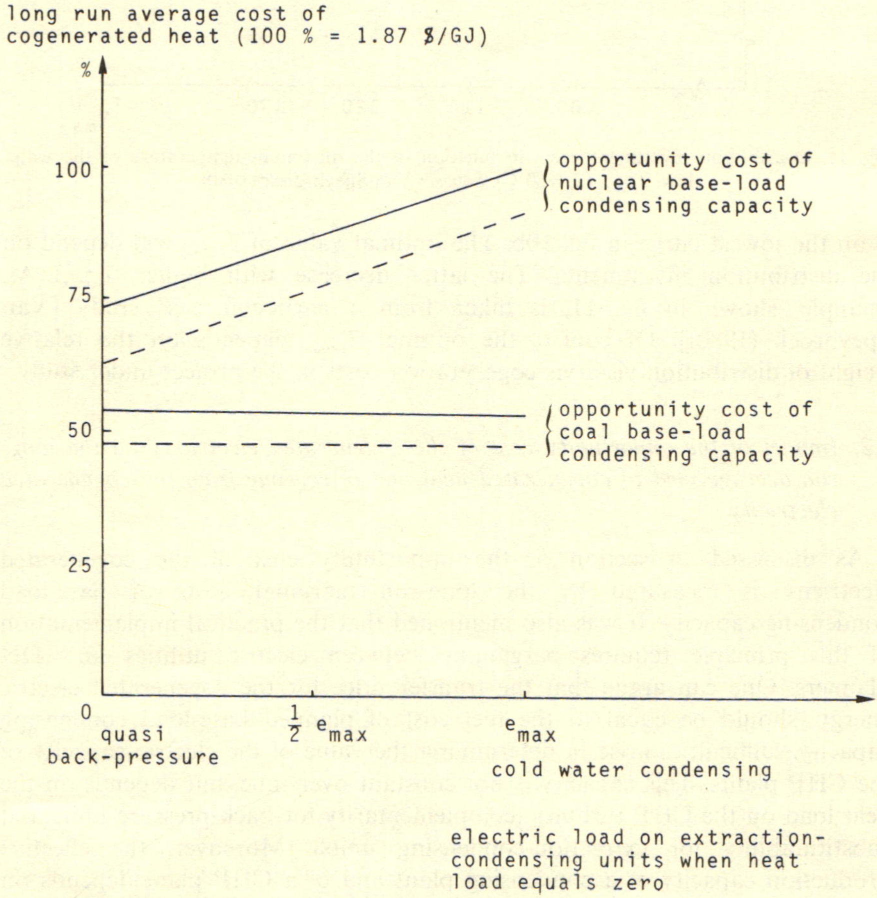

Fig. 12. Impact of the opportunity cost of the cogenerated electricity on the long-run average cost of cogenerated heat. No-stepwise heating —— and stepwise heating ----.

target for the extraction-condensing units when no heat load is available. Secondly, computations are carried out when electric utilities plan to expand their system with coal-fired condensing plants (table 3). Results are shown in fig. 12.

One observes that the long-run average cost of the cogenerated heat is significantly lower if coal-fired reference units are substituted for nuclear ones. Using the cost figures of table 3 and the operation modes implied, nuclear base-load electricity generation is cheaper than coal-fired generation. The gap between both narrows rapidly when the electric load on the extraction-condensing units during summer is lowered. When coal-fired condensing plants are used as reference plants, the results are not sensitive to this constraint because CHP units are also coal-fired. The steep slope of the two upper lines in fig. 12 shows that the operating conditions play a very important role when electric opportunity costs are evaluated on the basis of nuclear plants. One should keep in mind that the differences observed in electricity planning between nuclear and coal-fired base load capacity, should be reflected in analogous differences when the opportunity cost of cogenerated electricity is evaluated.

Finally, the distance between solid and dashed lines is larger when nuclear reference plants are used than when coal-fired reference plants are used. This is due to the higher weight put on guaranteed electric CHP capacity in the former case.

7. Conclusion

A model for evaluating the economic performance of CHP units in large-scale DH systems is presented. Analytical expressions for the gliding temperature operation mode are derived as simple functions of the load function $l(t)$. These together with the design variables of the DH system determine the temperature and mass flow functions. With these functions, and with the technical characteristics of the CHP units (i.e., data on the heat-electricity possibility set and on the energy-conversion efficiency of the CHP units), functions for evaluating heat output, electricity output and fuel consumption of the CHP units are derived. These functions (all polynoms in t) are integrated numerically over particular intervals of the time axis, resulting in energy quantitites (heat, electricity, fuel) required for assessing the economics of cogeneration.

This model is imbedded in a larger computer program for DH project appraisal studies in Belgium. Because all computations are based on analytical functions, the evaluation of an investment strategy demands little computing time. Therefore, the model can be used easily as an interactive system model of DH systems [Matheson and Howard (1968)]. As such, it becomes a valuable tool for DH planners of new large-scale DH projects, and for DH operators in their choice on the installment of a particular CHP turbine (see section 1.3).

In addition, the above description of the CHP production process and DH operation modes, provides the building blocks for the development of a program optimizing the operation of existing DH systems. The boundaries of the production-possibility sets of the cogeneration units (fig. 7) can be written as linear constraints on the outputs of the units. The fuel functions [(17) and (21)] allow to compute the corresponding fuel consumption. With (4), the relationship between temperatures and mass flow of the circulating water in the network is given, allowing the assessment of pumping costs. Also the storage of low-temperature heat can be incorporated when storage capacity is available.

This article was limited to the analysis of CHP production with steam turbines. It is possible to incoporate other CHP technologies in the provided framework [Verbruggen (1979)]. The model is imbedded in a program for DH planning but is also useful to DH operators. For a complete analysis of DH, the other components of the DH system should be added.

Appendix: Symbols

DH = district heating,
CHP = combined heat and power (cogeneration),
q, Q = heat capacity (MW), production (MWh),
e, E = electricity capacity (MW), production (MWh),
f, F = fuel flow (MW), stock (MWh),
t = time (in hours or 1000 hours),
T = temperature (in centigrades),
T_v = temperature of the DH water leaving the production plants,
T_r = temperature of the DH water arriving at the production plants,
$T_{v,max}$ = T_v at heat peak load conditions,
$T_{v,min}$ = T_v during summer.

Because the time unit equals one hour, capacity is in MW and energy in MWh, numerical values do not change by transiting from capacity to energy, or vice versa.

References

AGFW (Arbeitsgemeinschaft Fernwärme), 1981, Hauptbericht der Fernwärmeversorgung 1980, Fernwärme International 10, no. 6, 318–327.

BMFT, 1975, Einsatzmöglichkeiten neuer Energiesysteme, Programmstudie Sekundärenergiesysteme, Teil V: Fernwärme (Bundesministerium für Forschung und Technologie, Bonn).

BMFT, 1977, Gesamtstudie über die Möglichkeiten der Fernwärmeversorgung aus Heizkraftwerken in der Bundesrepublik Deutschland (Bundesministerium für Forschung und Technologie, Bonn).

Frilund, B., 1975, Turbines for power production in combination with district heating (Stal-Laval Turbin AB, Sweden).

Main, K., 1981, Combined heat and electrical power (CHP) in the United Kingdom, Fernwärme International 10, no. 2, 58–66.

Matheson, J.E. and R.A. Howard, 1968, An introduction to decision analysis (Stanford Research Institute, Standford, CA).

Mühlhäuser, H., 1977, Steam turbines in conventional combined district heating and power stations, Brown Boveri Review (Baden), Sept., 552–565.

OECD, 1978, Heat production and distribution (Organisation for Economic Cooperation and Development, Paris).

Reynolds, W.C. and H.C. Perkins, 1977, Engineering thermodynamics (McGraw-Hill, New York).

Scheid, F., 1968, Numerical analysis, in: Schaum's outline series (McGraw-Hill, New York).

Telson, M.L., 1975, The economics of alternative levels of reliability for electric power generation systems, Bell Journal of Economics 6, 679–694.

Turvey, R. and D. Anderson, 1977, Electricity economics: Essays and case studies (A World Bank Research Publication by The Johns Hopkins University Press, London).

Van Speybroeck, P., 1980, Schatting van de investeringen in het distributienet van stadsverwarming (University of Antwerp, Antwerp).

Verbruggen, A., 1979, Planning of district heating in Belgium, Ph.D. dissertation (Department of Applied Economics, University of Antwerp, Antwerp).

Verbruggen, A., 1980, District heating: Estimation of a standard load duration curve, International Journal of Energy Research 4, 385–398.

Zenker, W., 1981, Auslegungskriterien für die Dampfturbosätze eines Heizkraftwerks mit einer Leistung von 110 MW (elektrisch), Fernwärme International 10, no. 2, 39–45.