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DISTRICT HEATING: ESTIMATION OF A STANDARD LOAD DURATION CURVE

AVIEL VERBRUGGEN*

University of Antwerp, Thaliastraat 71, 2600 Berchem, Belgium

SUMMARY

District heating projects are planned with the help of a standard heat load duration curve, being the representation of the most likely load pattern the system will have to meet.

A systematic procedure to estimate the standard curve is set up: the daily average outside temperature density function is transformed with a relationship between this temperature and the daily average heat load, resulting in a density over the heat loads. The cumulative distribution of the latter is the day load duration curve.

Because the result has to be used for capacity planning, the unitary time interval has to be small, e.g. 1 hour. Therefore, the day curve is refined by incorporating the fluctuations over the day.

The hour load duration pattern is approximated analytically using the collocation polynomial technique of Lagrange.

Results are in agreement with the postulated standard patterns of the literature. The paper adds to the state of the art primarily by presenting a systematic procedure to derive the curve and by expressing the curve analytically. The systematic approach allows a clear interpretation and an extension of the varous steps of the load analysis.

KEY WORDS District heating Heating load duration curve estimation

1. INTRODUCTION

District heating is applied in Eastern Europe, Sweden, Finland, Denmark and West Germany. Because of the apparent benefits with respect to energy conservation, supply security, and environmental protection, interest in the extension and foundation of district heating systems revived in Europe after 1973.

In several European countries an appreciable R & D effort was commanded by public authorities in order to appraise the economics of district heating (e.g. BMFT, 1977). The construction of a district heating system is organized mostly with the help of a master plan, representing the least-cost layout of the production plants and distribution network at the completion of the project. In order to derive the optimal composition of the production park a 'standard' heat load duration curve is used (BMFT, 1977; Muir, 1973). The way this curve is derived is not mentioned in the literature.

In this paper, a procedure to derive a standard load duration curve is presented, and implemented with sample data referring to Belgium. The purpose of this research was twofold. First, it aimed at a comparison of the heat load structure in Belgium with those in other countries in order to verify the argument that the climate in Belgium is unsuited to the development of district heating. Secondly, the derivation of a standard heat load structure and its analytical representation are an essential step in the appraisal of district heating projects.

For this standard curve only load duration curves are considered, i.e. curves where the loads are ordered from highest to lowest values over the period. In this study the period equals one year, and the loads are expressed as hourly magnitudes. The heat load duration curve will thus be an ordered set of 8760 pairs (t_i, q_i), t_i being the hours of the year from 1 to 8760 and q_i being the corresponding heat loads (in MW). If $t_j > t_i$ than $q_j \leqslant q_i$.

* CIM doctoral fellow

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no observations are available. Examples of such features are: the hours people wake up or go to bed, the inertia of the occasional heat storage elements such as distribution pipes, buffer tank in the power plant, heated living rooms, etc.... Rather than with hourly figures, the study is carried out with daily average figures. The daily average hour load is calculated for each day as the arithmetic mean of the 24 observations. Because data for 30 days were incomplete or unreliable, the sample contains 1065 points. For this sample it is a plausible assumption that the average hour load during a day is only function of the relevant climatic conditions. All climatic factors are summarized by one variable, namely the average outside temperature during the same day. This simplification is necessary if one uses the methodology of this paper to derive a load duration curve, for it will be necessary to derive a density function of the weather variables. Fortunately, the reduction of the set of weather variables to one variable is no obstacle for the derivation of results accurate enough for the planning job (Hammer, 1974; Zühlke, 1978). The average outside temperature is calculated from hourly observations and transformed into 18°-days.

About the relationship between weather conditions and heat load little research has been carried out. Empirical curves measured by some district heating operators in Germany are shown in Figure 1. The decreasing marginal heat consumption with decreasing outside temperature is a generally identified property. (BMFT, 1975; Nelson, 1974).

A detailed analysis of the relationship is studied by Zühlke (1978). He includes in his regression equations the following independent variables: the daily average outside temperature, the average temperatures of the previous days, the number of sunshine hours and the wind speed. It is obvious that the generation of a density function incorporating all these variables is a difficult task. This task, however, is not attempted by Zühlke because his research aims only a derivation of a tool for ex-post control of the observed heat consumption. If one were to use the methodology of Zühlke for predictions, an 'equivalent' outside temperature variable, being a weighted combination of all relevant weather variables, could be used. The implementation of this procedure would require information about the synchronism of the weather phenomena and one can doubt if the final results will be significantly better for planning purposes.

Various specifications for the ($\bar{q}$–T) relationship were tested. The constant term is an estimate of the constant hourly consumption during summer when energy is used only for water heating. This result is due to the use of the degree-days explanatory variable. If average outside temperature surmounts 18°C, or in other words if the 18°-days of that particular day equal zero, it is assumed that no heating takes place. The use of the degree-day variable will also improve the regression results because variations in the temperature higher than 18°C do not affect the heat load.

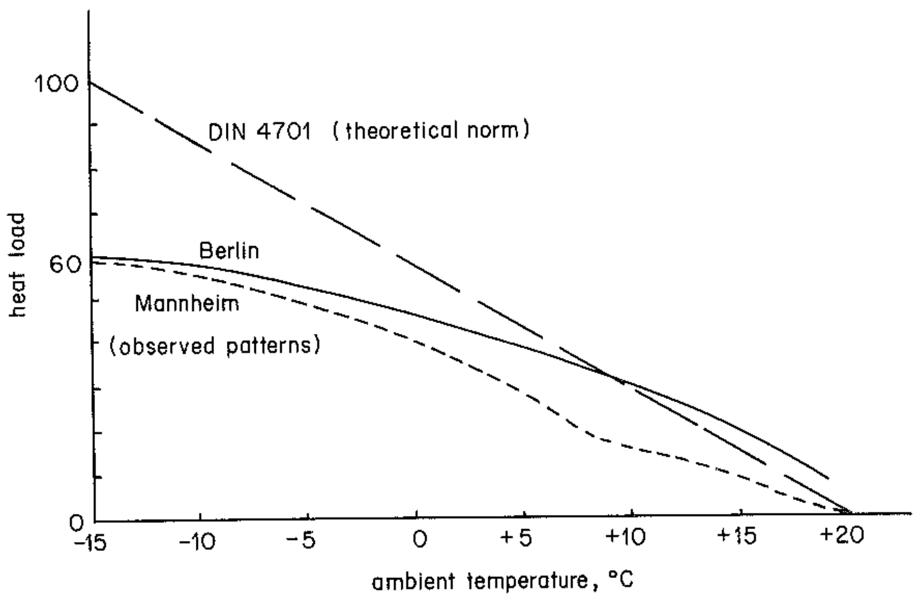

Figure 1. Heat demand as a function of the ambient temperature. From BMFT (1975), p. 103

Table I: Relationship between heat load and outside temperature (daily averages)

No.	Specification	n	R^2	σ
1	$\bar{q}=2{\cdot}0368+1{\cdot}0287x$ (0·0122)	1065	0·871	2·0744
2	$\bar{q}=1{\cdot}3233+1{\cdot}2978x-0{\cdot}0158x^2$ (0·0357) (0·0020)	1065	0·878	2·0167
3_a	$\bar{q}=1{\cdot}8232+1{\cdot}0637x$ (0·0127)	1040	0·871	2·0184
3_b	$\bar{q}=8{\cdot}8410+0{\cdot}5648x$ (0·2009)	25	0·256	2·1546
4_a	$\bar{q}=1{\cdot}6005+1{\cdot}1082x$ (0·0143)	964	0·861	2·0117
4_b	$\bar{q}=7{\cdot}7238+0{\cdot}6191x$ (0·0739)	101	0·415	1·6545

$\bar{q}$ = average hourly load during one day (MW), computed as the heat production of the day (MWh) divided by 24 (h)
x = number of 18° days of the day
n = number of observations used for a particular regression
R^2 = determination coefficient
σ = estimated standard error of the disturbance term.
The figures in parentheses below the estimated coefficients are the corresponding estimated standard errors.

To estimate this distribution one should use large samples of temperature observations. In this study 4015 daily average temperatures (11 years) serve as basis. The restriction to this 11 year sample is only due to the lack of more easily accessible data.

To generate the temperature distribution one has to abstract from the chronologic structure of the observations, or in other words the 4015 observations are to be seen as one random sample and not as the result of an experiment carried out 11 times and resulting in 11 observations with respect to a particular day. Using the latter approach, the oscillations are automatically flattened, and an obtuse bell-shaped pattern will be the result. If, on the other hand, all observations are approached as forthcoming from one probability distribution, the observed temperature oscillations are transferred into the analysis.

The problem of combining the outside temperature density function with the selected ($\bar{q}$–T) relationship, is a change-of-variable problem (Sullivan, 1977). Theoretically, this procedure can be used for any ($\bar{q}$–T) relationship and for any probability density function of the temperatures T. In practice, the computational burden will increase rapidly if no simple ($\bar{q}$–T) function and well-behaved density of T is assumed.

The density function $f(T)$ represents the distribution from which each year nature samples randomly 365 observations. Mankind could step into the foot-marks of nature, and sample from the same density function for each future year of his conceived district heating project. This procedure would result in as many load patterns as there are years to come. To derive a master plan, one restricts the analysis to one standard curve. To derive this curve, sampling has to be organized in a particular way, in order to obtain a set of 365 temperature values representing the normalized or most likely temperature set of any year. For this, one divides the area under the $f(T)$ curve in 365 equal areas and calculates 365 temperature values T_i corresponding to the 365 intervals. The set of T_i values is used with a particular ($\bar{q}$–T) function to derive a set of 365 $\bar{q}_i$ values.

A discrete approach rather than a direct analytical derivation of the load curve is used in this study for two reasons. The first is simple: if the study requires a load duration curve based on one hour as time unit, some transformation of the daily points into hourly magnitudes is necessary. In order to carry this out, an inverse of an analytically derived cumulative distribution function is required. In most of the cases an analytical formulation of the inverse function will be impossible. Even if it would be possible the transformation from a daily towards an hourly resolute, will require a transition from the continuous curve to a discrete approximation. At this point one is as far as with the discrete procedure. Secondly, the temperature density

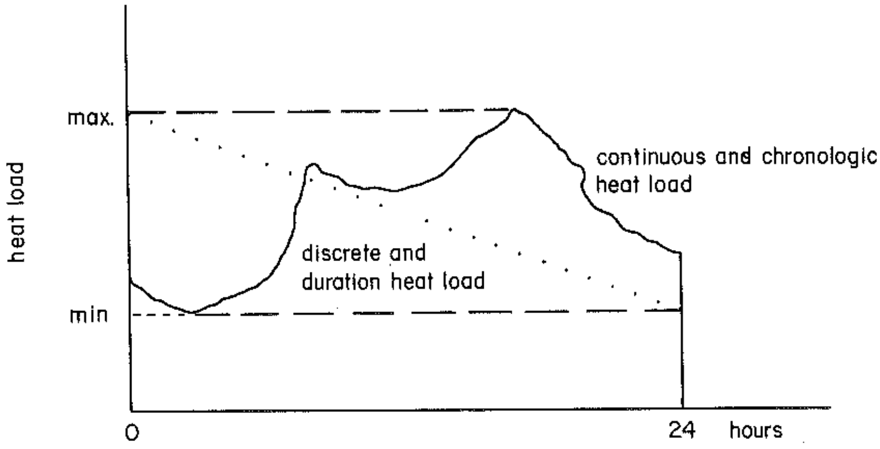

Figure 4. Heat load patterns during a day: schematic of the continuous/chronologic actual pattern and of the discrete/duration approximate pattern

information is lost in a duration function. The actual continuous, chronologic load pattern during a particular day is approximated by a discrete duration load pattern (Figure 4). This pattern is the result of a linear 24-step interpolation between the maximum and minimum heat load of the day. Randomly selected days were analysed in order to prove the appropriateness of such a linear approximation (Verbruggen, 1979). On basis of 350 observations in 1973, the relationship between the range R (=difference between maximum and minimum hourly load) and the daily average heat load is checked. A log–linear specification was the most suitable:

$$\log R = 0{\cdot}0403 + 0{\cdot}56042 \log \bar{q} \qquad n = 350$$

$$(0{\cdot}01651) \qquad R^2 = 0{\cdot}768$$

$$\sigma = 0{\cdot}2748$$

The elasticity of 0·56 indicates that the increase of the amplitude diminishes according to higher average loads.

The next step in the calculations aims at an ordered (q_i, t_i) set with $t_i = 1 \ldots 8760$, and q_i being the corresponding hourly heat load. The q_i values have to be ranked from highest to lowest values. If the q_i figures are generated in a random way one will have to rank 8760 numbers, requiring a lot of computing time. This time can be lowered substantially by generating q_i in the following way. The 365 daily average heat loads $\bar{q}_j$ $(j = 1, \ldots, 365)$ estimated with the $(\bar{q}\text{–}T)$ function and the normalized set of temperatures should be ranked from highest to smallest value. The hourly loads q_{kj} are computed from Figure 4:

$$q_{kj} = \bar{q}_j + \frac{R_j}{24}(13 - z)$$

and

$$k = 1, \ldots, 24$$

$$j = 1, \ldots, 365$$

$$z = k \text{ if } k \leqslant 12$$

$$z = k + 1 \text{ if } k \geqslant 13$$

$$R_j = 1{\cdot}0411 \bar{q}_j^{0{\cdot}56042}$$

The resulting q_{kj} values constitute a 365×24 matrix. The structure of this matrix is such that any element is smaller than or equal to all elements in the north-west corner, and larger than or equal to all elements in the south-east corner of the matrix with respect to the particular element. By using this structure of the q_{kj} matrix the transformation of this matrix to a ranked q_i vector $(i = 1, \ldots, 8760)$ is efficiently done.

Interval {1000–6000} hours

$$q_2(t) = 21{\cdot}0619 - 3{\cdot}1419t - 0{\cdot}0533t^2 + 0{\cdot}2348t^3 - 0{\cdot}0602t^4 + 0{\cdot}0043t^5$$

Interval {6000–8760} hours

$$q_3(t) = -67{\cdot}6077 + 36{\cdot}1931t - 5{\cdot}5564t^2 + 0{\cdot}2643t^3.$$

The load duration function $q(t)$ is a valuable tool in planning the production of district heating systems (Verbruggen, 1979).

6. DISCUSSION OF THE RESULTS

6.1 *Use of the results*

If one uses the load structure without any change three assumptions are implied. First the service area under study should represent a customer composition similar to the one of the sample, i.e. heat will be consumed for space and water heating by residential/commercial customers. Secondly, it is assumed that the daily average outside temperature distribution of the district under study coincides practically with the distribution observed (Figure 3). Thirdly, in the new project no peak load pricing is issued.

If none of these assumptions is valid the procedure has to be implemented all over again. If only the first and third stand, the derived ($\bar{g}$-T) functions remain valid, and if the representative T density is estimated, the procedure can be taken up from that point on. If only the climatic condition is not fulfilled, and if one estimates a new load pattern, one should not be surprised that the new pattern differs only slightly from the one presented here. The reason is that the temperature distributions of most places in the same climatic zone are identical, with the exception of a horizontal translation along the T-axis (Figure 3). Such a horizontal shift will affect primarily the necessary peak capacity, and only slightly the load structure itself.

The load duration curve, and the analytical formulation $q(t)$ are scaled on the estimated peak demand or heat capacity of the conceived district heating system. One proceeds as follows. The forecasted annual heat energy demand at the production plants (i.e. transport and distribution losses included) is divided by the standard production duration (i.e. 3240 hours, as will be shown in the next section). The result is the expected peak capacity necessary to meet any load. This figure divided by 28·47, the peak of the derived load function and only valid for the sample system, provides the scale factor with which the $q(t)$ function has to be multiplied. One disposes now of the most likely load duration pattern of the district heating system during the year the particular heat production is demanded.

6.2 *Comparison of the results with observations*

An ex-post confrontation of the results with the load characteristics of the sample system is desirable to evaluate the impact of the assumptions and approximations made. Some important load parameter values are summarized in Table II.

The expected production duration equals 3240 hours. Observed durations vary widely. This is due to the availability of a large margin of overcapacity. Normally, the fluctuations of the minimum outside

Table II. Annual heat production, peak proper, number of heating degree days and production duration: observed data and standard load curve.

	Number 18°-days	Annual heat production (MWh)	Peak proper (MW)	Production duration (h)
Observed data				
1972	3101	95,102	30·23	3146
1973	2984	90,717	23·26	3900
1974	2866	84,149	27·91	3015
Standard curve	3005	92,241	28·47	3240
Lagrange polyn.		92,290	28·47	3241

low outside temperatures and/or from oscillations around the ($\bar{q}$–T) relationship and/or from the demand fluctuations within a day. To formulate an optimal policy with respect to reserve provision, one has to estimate the probability that a particular capacity will be insufficient to meet the demand. The procedure is discussed with respect to the capacity $q_{max} = 28{\cdot}47$ MW. The probability that this (100 per cent reliable) capacity is too low to produce the load during one or more hours of any day is estimated.

Formally: let $P(q_i > q_{max})$ equal the probability of a capacity shortage. This is equal to $P(q_{kj} > q_{max})$, i.e. the probability that a particular hourly load k, during day j, exceeds q_{max}, which in its turn equals $P(\bar{q}_j + \frac{1}{2}R_j > q_{max})$, i.e. the maximum hourly load of that particular day exceeds q_{max}. This step in the reasoning implies that one estimates only the capacity shortage on a day basis. Irrespective of the duration of the shortage (one or more hours), the weight of the event remains the same. This approach is backed by the interdependency of sequential hourly loads. Finally, this probability equals $P(\frac{1}{2}R_j > q_{max} - \bar{q}_j | \bar{q}_j, T)P(\bar{q}_j|T)P(T)$.

The density function of T is derived from observed values over a long period. The probability that a particular temperature T^* will occur is defined as the ratio of the number of times this particular temperature is observed to the total number of observations. The stepwidth on the T-axis equals 1° Celsius.

The distribution of $\bar{q}$ is assumed to be symmetric around the value $g(T^*)$, i.e. the value of the ($\bar{q}$–T) function evaluated in point T^*. The probability that $\bar{q}$ equals exactly $g(T^*)$ MW is 0·5, and that it equals $\{g(T^*)+3\}$ MW, respectively $\{g(T^*)-3\}$ MW, is 0·25 in both cases. Analogous assumptions are made as to the probability distribution of R (see Figure 6).

Under the density functions an incomplete probability tree is drawn. This tree should be expanded from the lowest temperature in the sample to the temperature level at which the probability of a power shortage equals zero. Formally the temperatures to be studied have to satisfy the inequality

$$g(T)+3+\tfrac{1}{2}\{k[g(T)+3]+2\} \leq g_{max}$$

With the assumed density functions, and with $g_{max} = 28{\cdot}47$ MW, the temperature threshold equals $-2{\cdot}7$°C.

Some results of the discrete probability analysis are shown in Figure 7.

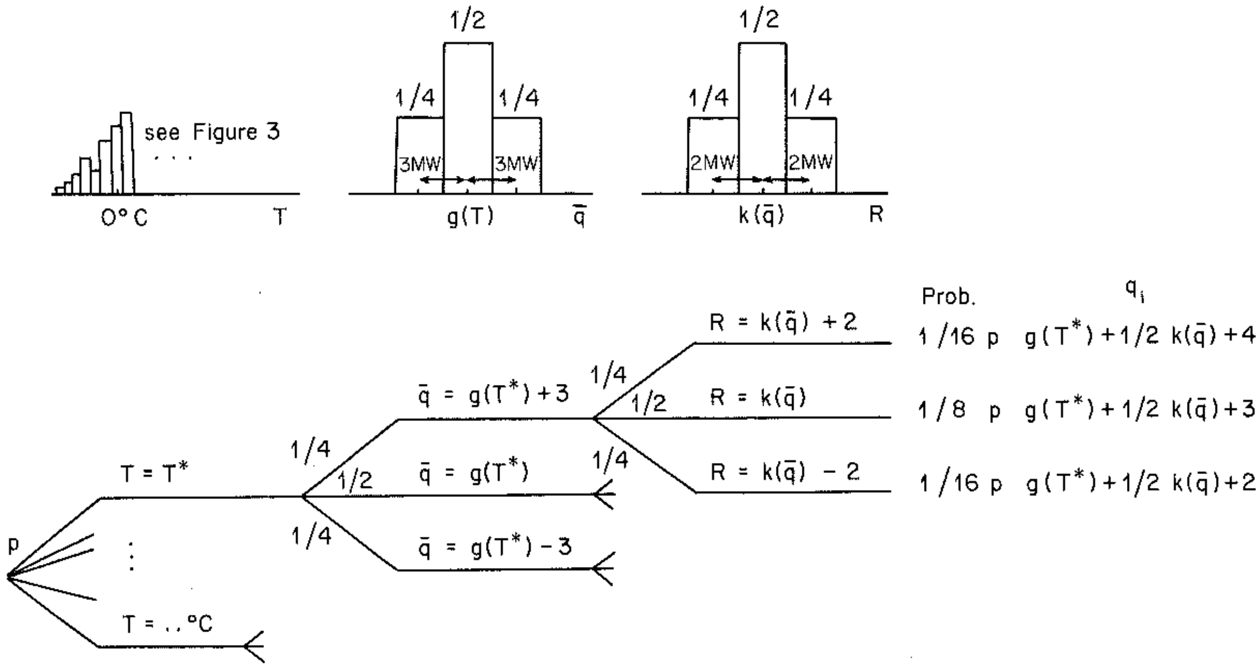

Figure 6. Probability of capacity shortage: schematic of discrete procedure

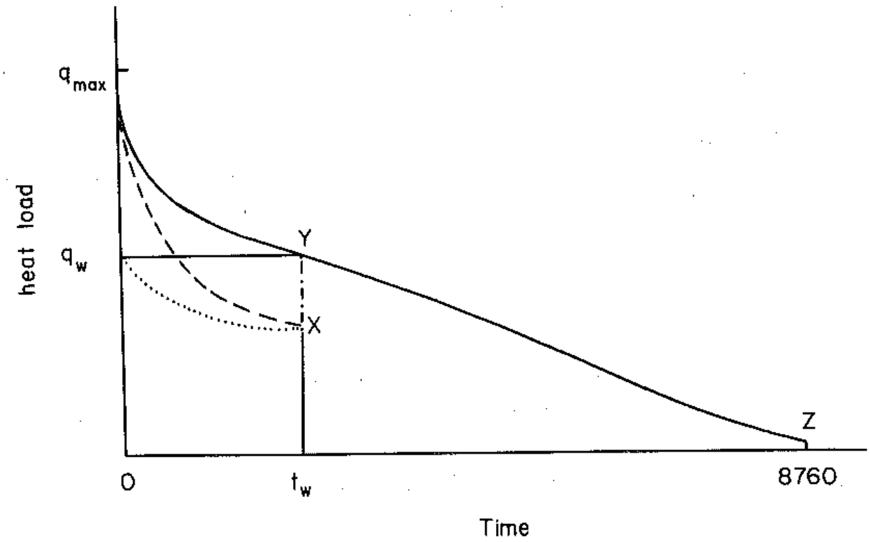

Figure 8. The standard load duration curve and load structures resulting from applying peak-load pricing

the intersections A → C and C′ → Z′. In order to derive the load structure q_wYZ, it is necessary to follow the intersections A → C′ and C′ → Z′. The decline in total capacity from q_{max} to q_w is based on the assumption that the demand curve AA′ is not a vertical line (zero price elasticity of demand). However, demand during peak hours may be quite inelastic. Peak-load pricing in this case would give rise to a load structure comparable to q_{max}XYZ in Figure 8.

Because of the particularities of heat load, peak-load pricing is not well suited to improve the allocative efficiency. This may be one reason why the problem is not discussed in the district heating literature. In order to keep the required production capacity down, two-part tariffs are applied in, as far as I know, all district heating systems. Because the system load $q(t)$ is the addition of individual loads $q_i(t)$, a two-part tariff can perform the task of a peak-load pricing schedule quite well. Before implementing the latter one, theoretical and practical research should be carried out in order to identify and avoid pitfalls as mentioned above.

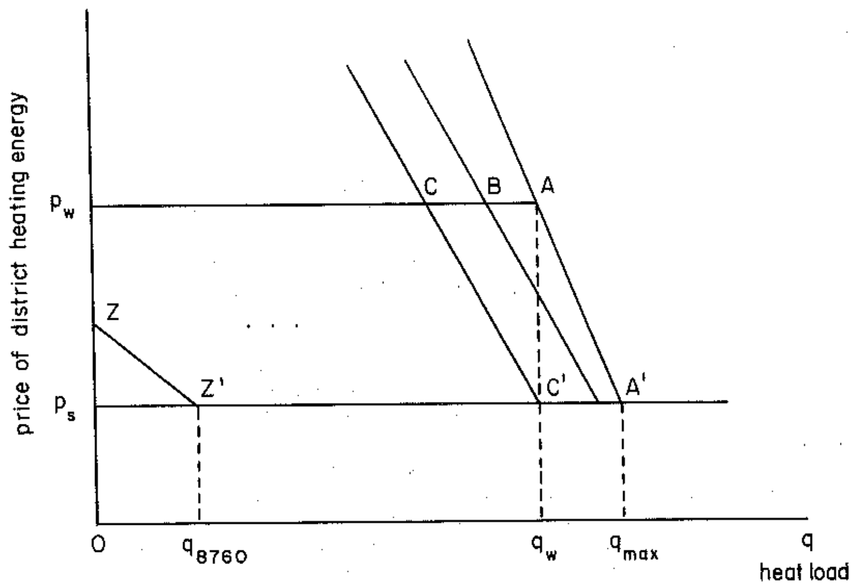

Figure 9. Peak-load pricing of district heating energy